Project no. TIP5-CT-2006-031415

INNOTRACK

Integrated Project (IP)

Thematic Priority 6: Sustainable Development, Global Change and Ecosystems

D1.3.6 The state of the art of the simulation of vehicle track interaction as a method for determining track degradation rates
Part 2 – High Resolution models and the level of validation generally

Due date of deliverable: 2009-06-30

Actual submission date: 2009-07-01

Start date of project: 1 September 2006 Duration: 36 months

Organisation name of lead contractor for this deliverable: Network Rail Revision [Final]

Project co-funded by the European Commission within the Sixth Framework Programme (2002-2006)		
Dissemination Level		
PU	Public	x
PP	Restricted to other programme participants (including the Commission Services)	
RE	Restricted to a group specified by the consortium (including the Commission Services)	
CO	Confidential, only for members of the consortium (including the Commission Services)	

Table of Contents

Glossary

Abbreviation/acronym	Description
BEM	Boundary Element Method
BRR	British Rail Research
CAD	Computer Aided Design
CDSM	Critical Defect Size Model
DFG	Deutsche Forschungsgemeinschaft Priority Programme
DIFF	Dynamik Interaction mellan Fordon och Fastban
E-B	Euler-Bernouilli
FEM	Finite Element Method
FRF	Frequency Response Function
FTSM	Flexible Track System Model
HR	High Resolution
LCC	Life Cycle Cost
LCPC	Laboratoire Central des Ponts et Chaussées
MBS	Multibody System
MD	Molecular Dynamics
RCF	Rolling Contact Fatigue
RSSB	Rail Safety and Standards Board Ltd
S&C	Switch and Crossing
VTI	Vehicle Track Interaction
WLRM	Whole Life Rail Model

1. Executive Summary

The purpose of the INNOTRACK project is to achieve a 30% reduction in the life cycle cost of the railway infrastructure while doubling passenger traffic and tripling freight traffic by 2020. Problem areas have been identified by various work packages and analysis software tools were employed to evaluate the benefits of the proposed improvement methods. This reports aims at presenting the state of the art of the tools designed to simulate the vehicle track interaction (VTI) and how they are used in determining track degradation rates. The report primarily focuses on the tools that were used in INNOTRACK, but also takes a more global view by including other relevant tools that could also have been used and which are relevant to the goals sets for this research program.

This is the second state of the art report on this topic, deliverable D1.3.2 (with the same title) focussed on strategic models with a more global overview of the railway system and economic outputs. This report on the other hand focuses on high resolution models, i.e. those having a more detailed mechanistic modelling approach. It also aims to provide information on the level of validation achieved from the various tools with regard to track damage prediction.

A reference table listing the various tools is included. Most of these are based on Multibody System dynamics (MBS) or Finite Element Methods (FEM) are reviewed in the 1st section of this report. Each tool, whether commercial or in-house, is described either from first hand user experience or from details obtained from the developer. Necessary input data, level of detail and possible outputs are given, with a particular focus on the respective limitations of the tools.

In a following section the Vehicle Track Interaction (VTI) tools are then linked to specific type of track degradation that they are able to predict: wheel-rail contact damage, track component degradation or formation deformation. Wherever possible an example of the type of analysis carried out within INNOTRACK or elsewhere. Reference to any validation carried out within INNOTRACK is included in the summary table of the tools.

The report finally provides some guidelines on the type of tools that may be used depending on the track degradation modes that are under investigation and that have to be quantified. Some conclusions on the link between the engineering data generated from the high resolution models and how these may be used for life cycle damage prediction also appears. A discussion on the respective roles of academics and industry on the subject of track damage prediction, particularly ballast behaviour prediction and its settlement, is included in conclusion.

2. Introduction

Railway engineering practices and relevant researches have traditionally been split around the wheel-rail interface which is as much a physical as it is a symbolic interface. Historically this interface defined the limits of certain disciplines resulting in separate software tools being developed for studying vehicle and track dynamics. The former have been developed based on Multibody System (MBS) dynamics codes and several well known commercial codes are available across the industry. Railway track models on the other hand are usually based on Finite Element Method (FEM) or Boundary Element Method (BEM) and are still being used mainly by researchers in universities. Despite this disciplinary division, both vehicle and track interact with one another and are subject to shared dynamic loading conditions and they both need to contribute to the same essential requirements: be safe, be reliable and be economical.

Due to the ever increasing computer hardware capabilities and a constant drive from the software industry to integrate together existing engineering analysis tools (e.g. FEM and MBS), the complexity of the systems that can be modelled also constantly increases and the limit between the two categories of tools becomes less distinct.

The present report aims to describe the software tools available according to the two broad categories mentioned above, and to provide the current state of the art concerning their use for the prediction of track degradation and their respective limitations. Several reviews exist in this field and this report does not intend to duplicate these. However the main capabilities and limitations of current vehicle-track interaction (VTI) tools are reviewed, keeping in mind the main technical challenges they face in terms of future development. Mechanistic models that are used together with VTI models for predicting track degradation are also presented. A list of the most relevant tools used in INNOTRACK or known within the project partners is then drawn in Table 1 with some comments on their respective capabilities/limitations and their level of validation.

The report is thus structured as follow:

- A list of existing state-of-the-art reviews and surveys on VTI tools (Annexe 7.1).
- Current review of VTI models.
- A list of track degradation modes included in this review and how each tool relates to them.
- A table of all the listed tools included in this report.
- A conclusion on the validation aspect of high resolution models, their usage for damage prediction and eventually for economical LCC calculation.

3. Background on vehicle track interaction (VTI) models

3.1 Existing state of the art reviews on vehicle-track interaction

A certain number of literature reviews and survey have been produced in the last 15 years on the subject of vehicle-track interaction modelling. In order to provide some background information and matter for discussion in this report, the most relevant references have been included in annexe 7.1 according to three categories, namely a) vehicle-track interaction in the mid-high frequency range [1, 2], b) multibody system dynamics techniques [3] and c) wheel-rail contact damage mechanism and track deterioration [4-9]. All reviews are fully referenced with hyperlinks to their electronic sources and a description of their content and conclusions.

3.2 Review of vehicle-track interaction modelling tools

Vehicle track interaction modelling tools either focus on the vehicle representation using Multibody System (MBS) dynamics techniques, or they focus on the track representation now mostly based on Finite Element Method (FEM). MBS are now mature software packages that are available commercially and are well validated. FEM based track models on the other hand tend to be developed by small groups or individuals within universities and validation data or benchmark of models is more rare. Both methods are briefly presented here for background information as well as some details on the modelling of the interface between the two: the wheel-rail contact.

3.2.1 The Multibody Dynamics approach for railway vehicle dynamics

Railway specific MBS codes were originally developed in the late 1970's by independent railway organisation for their own specific research. They can now include complex non-linear modelling features such as stick-slip dry friction dampers and sophisticated wheel-rail contact algorithms. The main commercial MBS packages are Vampire, Simpack, Nucars, Gensys and VI-Rail (previously called ADAMS/Rail) which are well validated for vehicle and wheel-rail contact dynamics and they participated to the 1998 Manchester Benchmarks which results are published by Iwnicki in [10] (summary are included in annexe 7.2). A number of reference books exist on railway vehicle dynamics including: [11] and [12]. Another more general reference book is [13]. Journal papers such as [14] , [15], [16], [17], [18] and [19] illustrate the utilisation of these tools with a specific focus on the wheel-rail contact.

The track input and track model

Track geometry data are usually used as input to the models. Data required include longitudinal level and alignment, cant and gauge irregularities and can be directly imported from track recording coach data after high pass filtering is applied to the recorded signal. As far as the track model is concerned, most packages provide a simplified representation of the structure onto which the rail is attached. Each rail is considered a rigid mass suspended on another one representing the sleeper/ballast that itself has a flexible connection to ground. This flexible mass system offers some movement to the rail in the vertical-lateral plane and is usually modelled independently underneath every axle of the vehicle model. Figure 1 shows a graphical representation of a typical vehicle on track model.

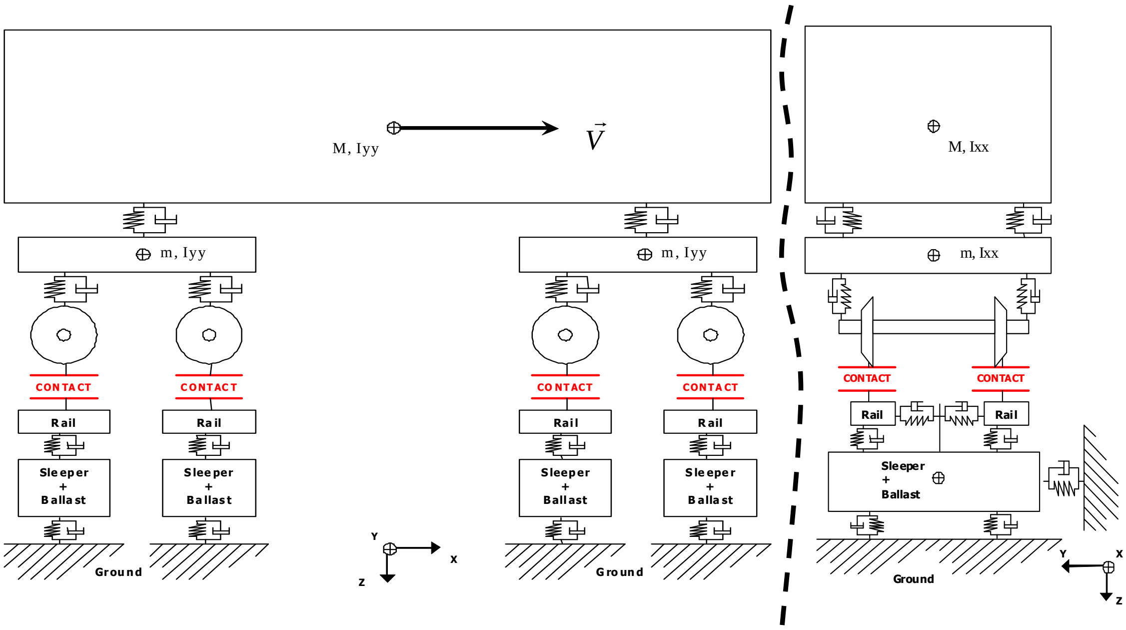

Figure 1: Typical vehicle-track model representation in MBS

The main limitations of this type of models are:

- No connection exists between the deformation of the track underneath one axle and adjacent axles.
- High frequency vibration effects of the rail such as bending and twisting are completely ignored.
- The track mass subsystem travels at the same speed as the vehicle model with potential inertia effects that have no real physical meaning.

Although the above disadvantages are generally accepted not to significantly affect the vehicle dynamic behaviour at low frequencies [20], they may have some impact on the prediction of the wheel-rail contact condition and forces which subsequently influence detailed wear and fatigue studies for a wider frequency range. In addition, the user is unable to determine the dynamic response of the track and its components for example, how much deflection is due to the rail-pad deformation or the sleeper movement, how much the rail bends and twists, how these differ along the track for various support conditions, etc.

Latest development regarding track models in MBS tools

In the last few years, some of the MBS software developers have incorporated more detailed track models that include:

- flexible components in the track, i.e. the rails and the sleepers
- flexible connections in the track, i.e. the pad-fastening system and ballast/sub-ballast interface

Flexible components such as the rail may sometime be defined in a 3rd party FEM software and transformed into a number of characteristic bending and twisting modes (eigen-frequencies and mode shapes) using a modal reduction method, and then imported within the MBS environment. Alternatively they can be defined directly within the MBS environment using built in multibody based flexible beam elements. Other flexible connections are directly taken from existing MBS capabilities to model linear and non-linear force-deflection and force-velocity based elements.

Examples of such track models include those presented by Zacher and Ambrogi [21] to study the interaction of a high speed train running onto a flexible bridge using ADAMS/Rail; the same method was further developed in VI-Rail to be used with the more advanced non-linear contact element to model different track forms such as conventional ballasted tracks and innovative slab track by Bezin

[22], [23], this led to the release of a new Flexible Track System Model (FTSM) plugin to the software; Dietz [24] also performed a vehicle on bridge interaction simulation using Simpack; Elkins [25] explored several techniques (FE being among them) using the software package NUCARS, and successfully applied this to the simulation of vehicles running over switches and crossings (S&C) with varying rail profile and track flexibility [26].

Wheel-rail contact algorithms in MBS

Section profiles of the wheel and rail shapes (typically measured using a MiniProf device) are included in the model for the contact calculation. Contact algorithms work out all contact conditions (lateral position, shape and size) based on the relative lateral displacement and angle of attack of the wheel with respect to the rail. Two steps are included in the calculation process:

(1) the calculation of the normal contact problem and

(2) the calculation of the tangential problem (or creep forces).

(1) The normal contact is based on Hertzian laws of contact fully described in [27], that assume an elliptical shape between the two non conformal rigid bodies. Other methods have also been developed to better approximate the contact in cases of rapid change in radii within the contact (e.g. flange contact), they re-use the Hertzian theory but apply it to series of juxtaposed discrete bands that form the contact patch. They are referenced as multi or semi-Hertzian contacts [28], [29], [30]. Figure 2 shows the results of a non-elliptical contact search algorithm.

(2) the tangential forces theories where first established by Carter [31] and later fully described by Vermeulen and Johnson [32], but the main contribution was made by Kalker [33] with the code 'CONTACT' which is based on the 'exact' theory. However it is too slow to be implemented in a MBS environment. Kalker derived a faster 'simplified' theory called FASTSIM [34]. A recent benchmark of contact models used in various MBS software was undertaken at Manchester Metropolitan University and initial results are published in [35].

Figure 2: Multipoint contact with non-elliptical shape for a UIC60 rail and s1002 wheel in ADAMS/Rail - MEDYNA, from [36]

Limitations of current MBS contact routines

Common assumptions made by all routines are:

- Rigid wheel and rail profiles are assumed: the geometrical shape of the rail and wheel cross sections assume no deformation during the simulation[1].
- Constant linear-elastic material properties: constant values of Young's modulus and poisson's ratio are used for the wheel and rail.

These assumptions are of concern if one is interested in deriving damage prediction from the contact data (patch(es) size and shape and on the contact forces) where material as well as geometrical non-linearities should be taken into account. Comparison with FE models have helped to identify these limitations as presented by [37] and [38]. FE models certainly offer an advantage in terms of accuracy; however they also have their own limitations as explained in section 3.2.3.

Another limitation of some of the contact algorithms is the use of a pre-computed contact table for the contact condition, which are not capable of identifying multipoint contact and sometimes fail to detect the details of two point contact (on flange and on tread) due to the interpolation of the contact table. More advanced and more accurate methods calculate the contact conditions during the simulation based on the exact position and velocity of the wheel and the rail. However they are more time

[1] The Hertzian theories described above effectively calculate a penetration value of the two rigid profiles to approximate the real relative position of wheel and rail bodies under local deformation of the profiles.

consuming and are only relevant for specific applications: multiple contact, heavy flange contact or changing rail profile. This is why the main motive for developing these algorithm was to apply them to switch and crossing simulations [39], [26], [40] and [41].

3.2.2 Finite Element Methods for railway track modelling

Some advanced railway track models are based on the Finite Element Methods (FEM) and can include representation of the complete track system: the rails, the sleepers, the ballast and the subgrade. Models may represent several meters depth of track over distances of a few tens of meters in length. Discrete ground layers with various material properties may be modelled that interact on top of one another. A typical model requires extensive computation time and therefore only a simplified vehicle input load is typically employed. This is one reason why these models are mostly limited to vertical dynamics on tangent track.

Early models

Early beam theories such as Euler-Bernoulli (E-B) and work from Timoshenko [42] (including shear deformation and rotary inertia terms, offering better accuracy at higher frequencies) form the basis for a lot of flexible track models. Beam equations were first applied onto an elastic foundation of infinite length (Winkler foundation), see equation (1) where the vertical deflection w at a distance x from the applied load q is expressed as a function of the beam bending stiffness EI and the track foundation coefficient k in [N/m/m], as illustrated in Figure 3.

$$EI \frac{d^4 w(x)}{dx^4} + k.w(x) = q \tag{1}$$

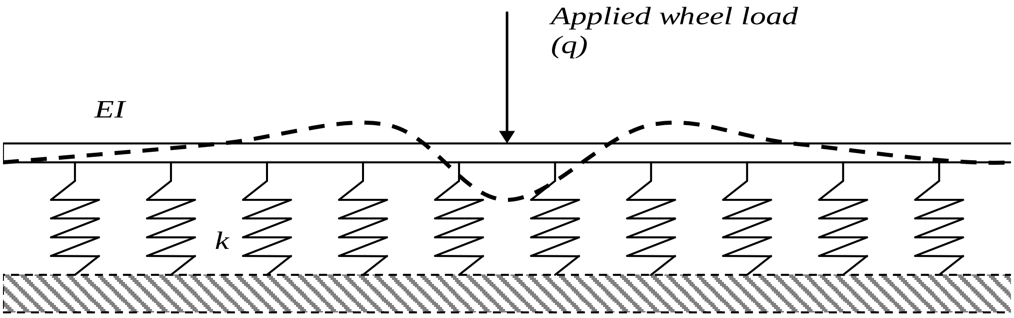

Figure 3: Deflection of an Euler beam on a Winkler elastic foundation (dashed line shows the deformed shape)

Simply supported beams, see Figure 4, have also been used to study vehicle-bridge interaction forces and deflection, [43].

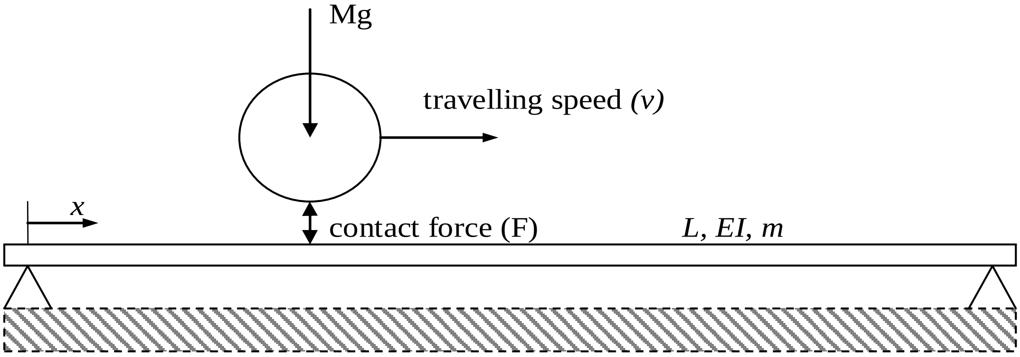

Figure 4: Simply supported beam with travelling mass

The continuous elastic foundation can be replaced with discrete layers that represent individual sleepers and ballast mass, see Figure 5. They can include several layers of the track structure and every mass has a flexible connection to the next layer to provide stiffness and damping for the system. Elasticity can be included between adjacent ballast masses to account for the ballast compression spreading along the track away from the load.

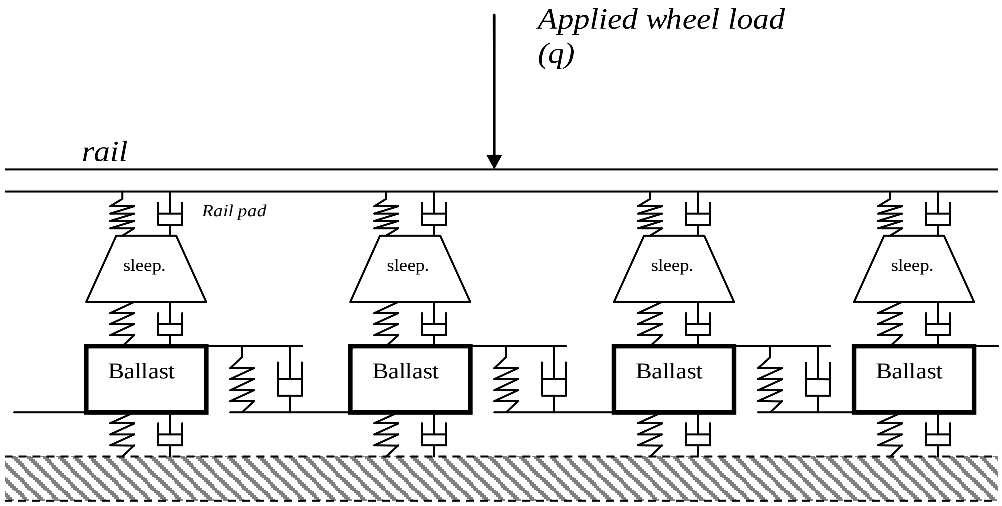

Figure 5: Two layers discretely supported ballasted track

Typical classification of flexible track models in recent reviews

As seen in reviews listed in annexe 7.1, vehicle-track interaction models have generally been classified into two categories: models that are based on (1) frequency dependent or (2) time dependent solution techniques.

(1) **Frequency** domain solution techniques are analytical tools that establish the relationship between the receptance of the track according to a unit force displacement of varying frequency.

<u>Advantages</u>: they are fast to run (mainly used for higher frequencies study: noise related issues).

<u>Disadvantages</u>: properties must be linearised (unsuitable for discrete events - rail joints, hanging sleepers, varying ballast stiffness, etc).

(2) **Time** domain solution techniques make full use of available FEM software or can be coded into scientific tools. They use time integration techniques. Can use modal superposition method to increase the speed of the model without significant loss of accuracy.

<u>Advantages</u>: non-linear properties or discrete events can be included.

<u>Disadvantages</u>: Take longer to run.

Benchmarking of railway track dynamic models

A couple of recent benchmark comparing various railway track dynamic models [44, 45] were published by researchers from Queensland, Australia. The first benchmark exercise was purely theoretical and made an attempt at comparing the results from various models listed hereafter: DARTS (DynTrack System, USA), DIFF (CHARMEC, Sweden), NUCARS™ (AAR, TTCI, USA), SUBTI (Technical University of Berlin, Germany), TRACK (Stuart Grassie Engineering solutions, UK) and VICT (Southwest Jiaotong University, China). The exercise involved simulating a vehicle negotiating a dipped rail joint and comparing responses at the wheel-rail contact as well as from the track (rail and sleeper). Flexible sleepers were included in most models so that bending moments in the sleepers could be compared. Rails and sleepers were modelled as Timoshenko beams in most models and the ballast based on Winkler or half space assumptions. A fare comparison was made difficult because every party involved modelled the dipped joint geometry in different ways due to the lack of given specification.

For the second benchmark published, two participants withdrawn: TRACK and VICT and two additional participants joined in: DTRACK (DynTrack System, Canada USA) and VIA (University of Valencia, Spain). This time a measured longitudinal rail profile was provided to all participants and site measurement data was available for comparing with the numerical models predictions. Compared data were wheel-rail contact force, rail acceleration mid-span between sleepers, rail shear force and sleeper bending moment in two locations. Despite a detailed set of input data and run conditions, no participants were able to reproduce results that were consistently comparable to the field data or other models. However NUCARS performed well overall, and interestingly it is the only tool originally developed for vehicle dynamics, and DTRACK was the only one with no substantial differences with field data and other models.

It seems from the above benchmarks results that track dynamic models still need further improvement in order to achieve comparable results with field experiment. More benchmark of the type carried out above are encouraged to achieve this goal and make track models more widely accepted and standardised.

Limitations

Because of the high level of details that track models can achieve, compromises are often made:

- Symmetrical track: lateral vehicle and track dynamics is ignored
- Wheel-rail contact represented by vertical Hertzian spring law
- Simplified vehicle load: most often a single sprung mass with additional harmonic force.

Typical track behaviour

Track models are often compared and calibrated against receptance graphs measured on real track. They inform on the stiffness behaviour of the track (vertical axis is the inverse stiffness in m/N) across a range of frequencies. The example below from Ripke and Knothe [46] shows the main vibration modes of the track identified by peaks and troughs and measured a) above a sleeper and b) in between sleepers, as follows:

❶ 1st peak – rail bouncing on ballast in phase with sleepers – around 150Hz

❷ 2nd peak – rail out of phase with sleepers bouncing on ballast – around 450Hz

❸ 3rd peak in configuration b) – rail pin-pin bending resonance frequency – around 1,000Hz

❹ 3rd peak in configuration a) – rail pin-pin bending anti-resonance frequency – around 1,000Hz

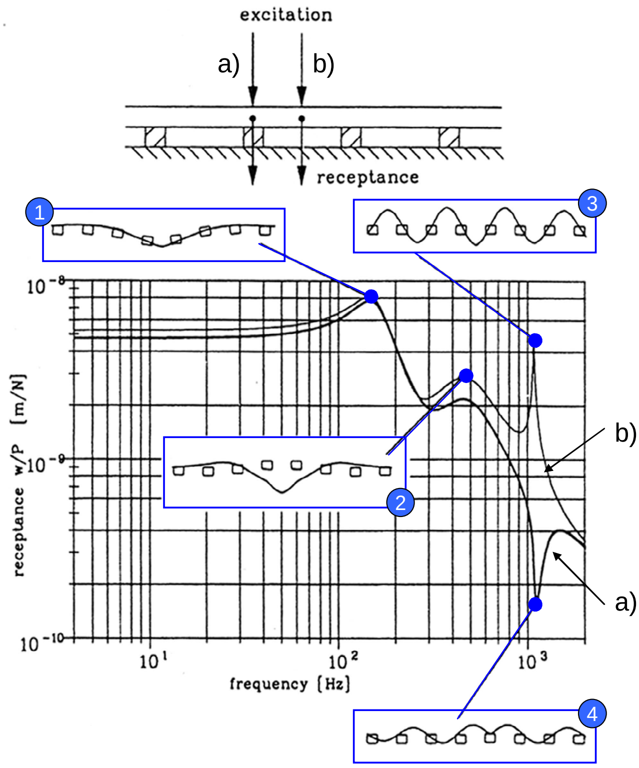

Figure 6: typical track vertical receptance graph from Ripke and Knothe [46]

Similar graphs may be obtained for the lateral track behaviour as discussed by Nielsen, Lunden et al. in [8], showing lower frequency response and the additional influence of the rail torsion modes at higher frequencies.

Advanced 3D vehicle - flexible track models

More detailed 3D models of the track system have also been developed in recent years, taking advantage of available commercial FE software capabilities. Several discrete layers underneath the rails such as sleeper, ballast, sub-ballast and substructure can be included and complex material laws which govern the track foundation behaviour under cyclic loading can be included. Even the rail may be built using 3D solid elements. Their main focus is the study of track settlement, transition zones and general dynamic interaction. Some examples are given here:

Lundqvist and Dahlberg [47] studied settlement using the FE software LS-DYNA. One half of a thirty sleeper section of track is modelled (assuming the track symmetry about its centre) with one wheel running onto one rail. Wave propagation from the axle load into the track structure is analysed and permanent deformation of the track with repeated wheel passage can also be predicted. The same model is also used to study hanging sleepers and their influence on track settlement [48].

Lane [49] includes a full rigid body train model running onto a complete track-subgrade FE structure to look at elastic wave propagation and possible mitigation solutions. The wheel-rail contact is handled as a linear-elastic spring and damper element in the vertical and horizontal plane.

Kabo et al. [50] illustrate how vehicle-track models with higher frequency capabilities such as DIFF may be used in conjunction with fatigue prediction tools such as FIERCE to look at short track features: e.g. rail joints. They show that studying the dynamic effects at such short wavelengths requires higher frequency capabilities only available with the use of detailed flexible track models.

3.2.3 FEM for wheel-rail contact analysis

Another FEM application focuses on modelling the wheel rail contact, providing accurate prediction of the contact patch shape and stress distribution at the surface and within the material. Telliskivi et al. [37] and Jaiswal et al. [51] show that stress may be overestimated in cases where the radii of curvature of the bodies in contact become small with respect to the size of the contact patch. When compared with CONTACT or Hertz theory, Wiest et al. [38] show with the example of a wheel on a crossing nose (S&C) that after including elasto-plastic material property, the contact area becomes larger (+50%) and the contact pressure distribution becomes lower (-42%). FEM is a useful complement to the conventional dynamics calculation method as it can represent the local non-linearities in the material as well as geometrical non-linearities. Although this method is very accurate[2] it is not yet applicable to a dynamic calculation of the vehicle-track interaction because simulating one contact configuration may involve hours of simulation.

3.3 Conclusion of key features and outputs of VTI models

As a conclusion to the above review of vehicle-track interaction models as a method for determining track degradation, it can be said that they are designed to predict the dynamic behaviour of the track subject to vehicle loading and they are able to produce physical quantities such as:

- Body displacement, velocity and accelerations: e.g. of the rail, of the sleeper, etc.
- Wheel-rail contact condition: e.g. number, location and shape of contact patch(es); normal and tangential forces for every contact.
- Forces within track: e.g. forces within rail pad/fastening, forces between sleeper and ballast, etc.
- Strains and stresses within track components or track layers: e.g. rail bending stresses, sleepers bending stresses, ballast pressure, etc.

These physical quantities can then be re-used as input to track damage models, external or embedded within the VTI model. The type of damage they apply to and how the VTI outputs are used is explained in the next section.

[2] FE models for the analysis of the wheel-rail contact condition also have their own problems and limitations. For example the penetration between the two bodies is unavoidable using the FE approach and it can have major consequences on damage prediction (c.f. deliverable 4.3.5).

4. Track degradation mechanisms and prediction methods

D1.3.6 focuses on high resolution models capable of predicting track degradation. The flow chart in Figure 7 gives an overview of the classification of the types of degradation that are likely to be assessed with such models. Three main types of track degradation are identified, each one requiring specific capabilities or input data from the HR vehicle-track models:

1. Rail damage
2. Ballast and subgrade degradation
3. Track components degradation (e.g. pad, clip, baseplate, underpad, etc)

The latter category will not be described further within this report because it is fully dependent on component design which varies a lot throughout Europe. Such activities revolve around fatigue testing using a combination of laboratory experiments, CAD and FEM software applications and are usually unpublished. Although fatigue analysis of components usually is independent from vehicle-track interaction simulations, VTI models may be used to provide more accurate input data on the service conditions experienced by these components when measured real service life data is not easily obtainable.

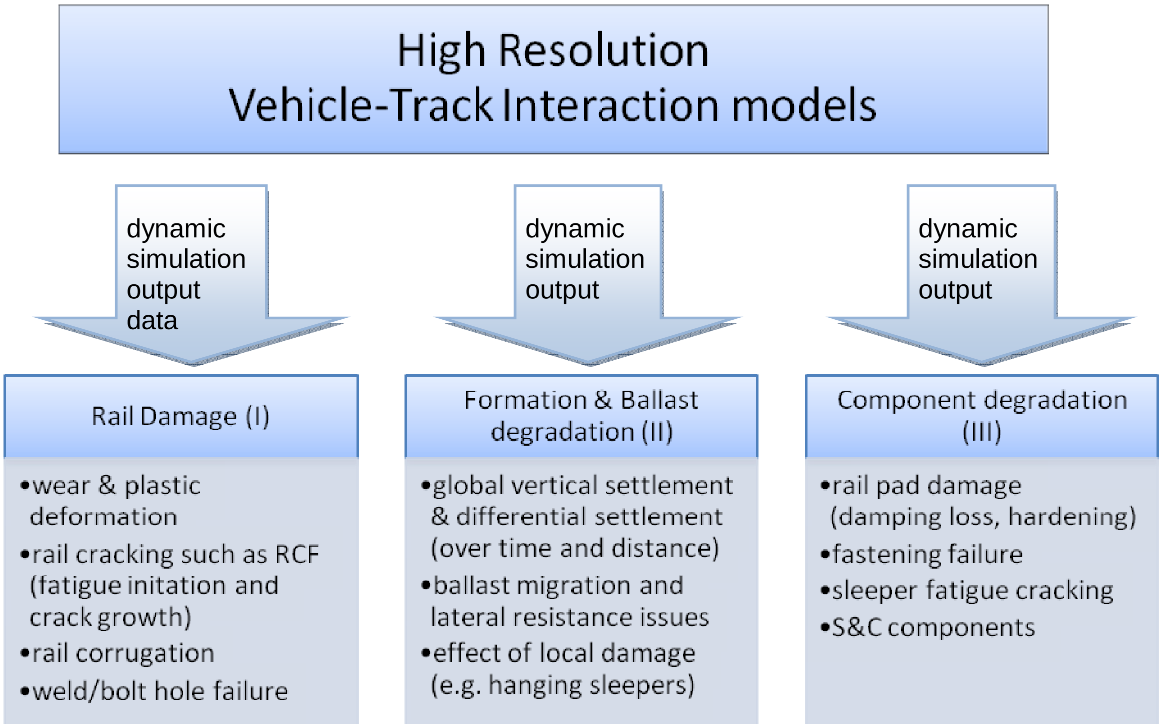

Figure 7: Type of track damage that may be studied using VTI models output

The methods and the links with vehicle-track interaction models for the category 1 and 2 are described here after.

4.1 Rail damage

Wheel and rail are subject to various types of damage mechanism that are governed by the normal contact pressure and the shear stresses within the contact patch(es)[3]. VTI models are therefore used to predict the contact conditions under various services configuration:

- Location of the contact patches and their shape on the section profiles
- Normal contact force for each contact patch
- Creepages within the contact patch in the lateral, the longitudinal and spin direction
- Corresponding creep forces and torque

This data is usually produced for the simulation of a vehicle on specific routes (equivalent to hours of running condition) while the damage mechanism develops over months or millions gross tons of traffic. There is therefore a feedback loop process taking place to exchange the data between the VTI model and the damage models at specific point in time. Criteria may be a certain amount of material remove or a certain mileage covered by the vehicle. This process is illustrated in Figure 8.

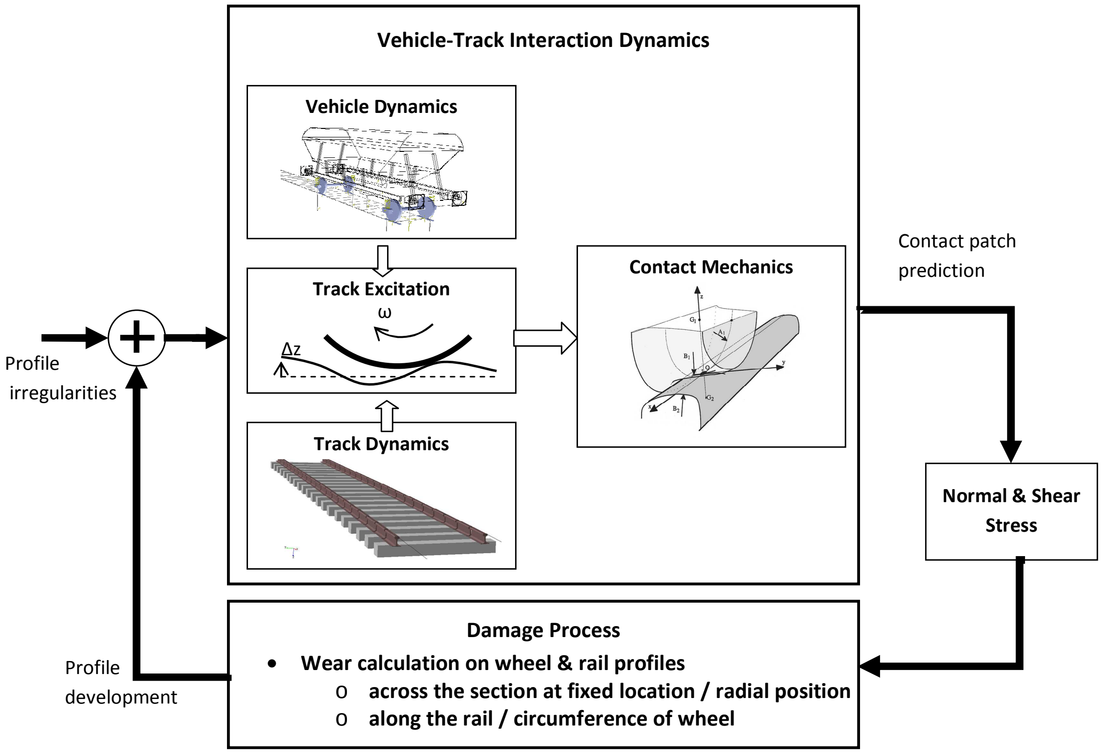

Figure 8: Integration of VTI dynamics and damage process in a long term feedback loop

The contact data from the VTI model is therefore used as input to the damage models. A number of these models exist and some of the main methods traditionally used in combination with VTI models are described here according to the two main types of damage they apply to: wear and rolling contact fatigue (RCF).

4.1.1 Wear

Wear occurs on both wheel and rail as a result of the relative velocity difference of the two contacting bodies in the contact zone, where part of the contact is in adhesion and the rest is sliding. There are

[3] It has to be noted that for a same normal contact pressure and shear stresses, different materials react differently; therefore the material type is also a governing factor.

currently two main computational methods[4] used to determine the wear on the rail surface from the output of multibody dynamic simulations:

The energy dissipation method

The amount of energy dissipated in the contact, known as $T\gamma$ (T-Gamma), is worked out from the sum of the products of creepages and creep forces for the longitudinal, lateral and spin components equation (2). This is a standard output of most railway MBS software, however the spin creepage, which is relatively small compared with the other terms, is ignored in most of them. $T\gamma$ is sometimes used to calculate a wear number taking into account the contact patch area A : equation (3), see for example [52].

$$T\gamma = \left(T_x \gamma_x + T_y \gamma_y + T_z \Omega_3\right) \quad (2)$$

$$W_n = \frac{1}{A}\left(T\gamma\right) \quad (3)$$

Experiments undertaken by BR Research [53] have also interpreted wear versus Tγ using a non-linear two stage relationship with mild wear occuring for low values of Tγ and severe wear linearly increasing for values above around 200N. See for example Figure 9 defined by Harvey and McEwan [54].

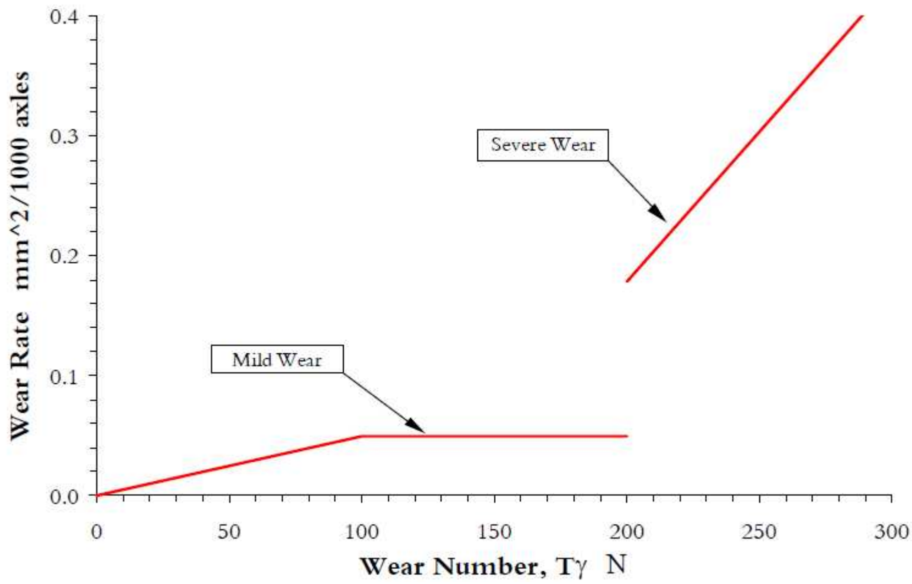

Figure 9: Relationship between wear number and wear rate for rail wear used by Burstow et al. [55]

The sliding wear method

The second method more widely used by tribologists is the Archard wear model [56]. The volume of material removed is calculated from the normal force, the sliding distance and the material hardness property, equation (4).

$$V_{wear} = k\frac{Ns}{H} \quad (4)$$

Where:

- V_{wear} = volume of wear
- s = sliding distance
- N = normal force
- H = hardness of the material whose wear is being calculated
- k = wear coefficient (calculated for a specific material combination)

[4] Although it may be argued that several other variations of these methods may be used across Europe.

The wear coefficient *k* is a function of slip velocity and contact pressure and is empirically identified by means of laboratory experiments. Example and validation of this model in conjunction with MBS for wheel wear are available from Jendel [57, 58] showing generally a good agreement between the predicted wheel evolution after 200,000km and the measurement (flange height, thickness and angle were compared). Bevan et al. [59] predicted the evolution of a new design of an anti-RCF wheel profile which were later successfully matched to real life trials measured ones. These examples highlight the fact that successful validation of this damage function exists only for the wheel rather than the rail. This fact may be attributed to the fact that the evolution of wheel wear on a specific vehicle may be predicted with better accuracy (as the route and mileage covered by the vehicle are well known) rather than for the rail at one specific location, for which precise traffic information may be harder to obtain and more uncertainties are introduced into the model (vehicle models, wheel profiles, lubrication. For example Enblom and Berg [7] used the same wear method for rail evolution with some success but they showed that overestimation against mileage was present and that lubrication efficiency was underestimated.

4.1.2 Rolling contact fatigue (RCF)

VTI models are currently principally used to predict crack initiation on the rail surface as opposed to crack propagation within the material which require alternative models based on fracture mechanics, e.g. [60]. These are usually very detailed ‘high resolution’ FE based models and are used independently from VTI models, although they can also be used in combination with VTI models so that they reuse predicted input data for wheel and rail respective position and loading forces. Another type of model called ‘brick model’ has also been developed and can be used for predicting crack initiation, e.g. [61]. They also rely on a discretisation of the two bodies in contact, although unlike FEM the elements are independent from one another, and the condition of each element is characterised based on whether or not they reach their critical shear strain as ‘healthy’, ‘weak’ or ‘failed’. In the later case they are considered as wear debris and are remove from the model.

The present review concentrate on the use of VTI model, therefore on the prediction of surface crack initiation based on the contact information predicted under various vehicle-track system conditions, e.g. vehicle type, curve radius, cant deficiency/excess, coefficient of friction, rail and wheel profile conditions, etc. Two main methods are commonly used in combination with MBS simulations to predict surface initiated cracks, the shakedown theory and the energy method based on Tγ.

Shakedown plot

The introduction of the shakedown theory below is largely taken from Foletti and Desimone [62] in which further details and references can be found. The shakedown theory originally proposed by Johnson and Jefferis [63] and fully detailed by Ponter et al. [64], is used to assess a materials response to three-dimensional contact loading. The so-called Shakedown Map has been created to determine whether the material response is fully elastic (no residual stresses or strains) or elastic-plastic (residual stresses and permanent strains can arise). A shakedown limit has been defined between these two conditions which describes the relationship between the normal wheel-rail contact pressure and the tangential shear stress which are likely to accumulate plastic strain and ultimately rail surface damage. Beyond the fully elastic response, three behaviours are identified:

- elastic shakedown: where the elastic limit is reached in the first few cycles but the steady-state is entirely elastic (the maximum load for which elastic shakedown can be achieved is known as the shakedown limit);
- cyclic plasticity or plastic shakedown: where the steady state consists of a closed cycle of plastic deformation; and finally
- Ratchetting or incremental collapse: where the structure accumulates increments of uni-directional plastic strain, leading to collapse.

The shakedown limit has been theoretically calculated from tests on different material properties and contact conditions with some degree of artificial ‘tuning’ to match the behaviour of typical rail steels observed in-service. The shakedown method may also be used alongside a wear model, to take into account the effect of wear in RCF growth, and to determine which degradation mode dominates. This relies on careful validation against empirical data to tune both models. Further example of a successful application of the shakedown theory can be found for example in Ekberg et al. [65] for wheel RCF.

Energy method based on T-gamma

The energy expended in the wheel/rail contact expressed as Tγ (see equation 2 in section 4.1.1 on wear) has also been used to express RCF damage. Wear and RCF are two separate phenomena that nonetheless have some influence on one another. A weighted function taking into account the summation of these two phenomena, one for wear and the other one for fatigue, was thus developed by Burstow [66]. Figure 10 shows a graph of the weighted Tγ function with the Tγ value on the abscissa and the resulting damage function on the ordinate. Three stages can be observed with RCF only at low energy dissipation between 15 and 75N, a zone of combined wear and RCF for values in between 65 and 175N and above that the wear regime becomes predominant by removing any RCF crack before they can become significant. This method has been used extensively in the UK and effort where spent in developing it as part of an asset and risk management tool: the Whole Life Rail Model (WLRM) [55]. A similar approach was also applied to the fatigue initiation on wheels for example by Tunna et al. [67]. It has to be noted that the validity of this method highly relies on the validation of the predictions against empirical data obtain specifically on UK track conditions. It is crucial that for such method to be successful, detailed and good quality measurement data are available. The method as currently used in the WLRM for UK track condition would not be directly applicable to other networks. For Tγ values below 15N no damage is predicted at present but it was argued that some level of wear is present in this situation which by not being taken into account may be responsible a general over-prediction of RCF in places. A new research programme by RSSB in the UK aims at improving the damage function in this respect.

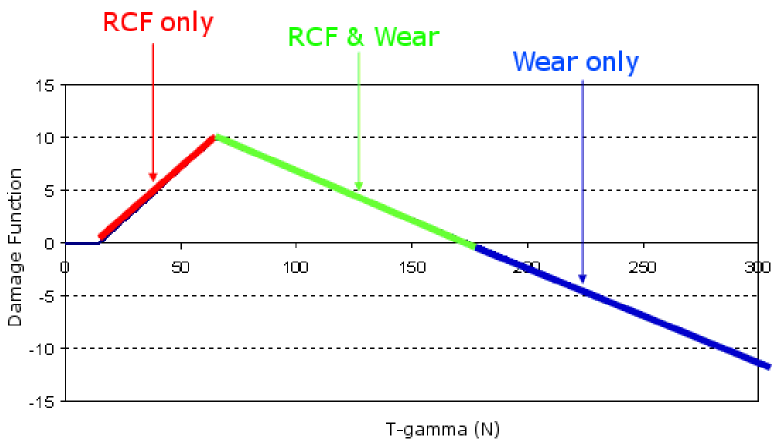

Figure 10: Weighted T-gamma function used in WLRM

4.1.3 Consideration for further damage mechanisms

The energy based methods presented above at the moment only considers fatigue and wear types of damage and there is a scope for extending the modelling to include further types of damage, for example plastic deformation, which can have a significant influence on the determination of wear and fatigue in the presence of high contact stress (e.g. flange contact). At the moment this is achieved by other methods such as detailed contact FEM based models e.g. Ringsberg et al. [68] or other 'brick models' [61].

4.2 Formation and ballast settlement

VTI models are able to predict the dynamic axle loads, the essential source of ballast and subgrade damage being transferred down to the track. These vertical forces may be used by damage models designed to work out the settlement of the track. Numerous settlement models have been proposed by many researchers and they were recently reviewed by Dahlberg [4].

4.2.1 Description

Most of these models consist of a mathematical equation similar to equation (5) that defines the vertical rail level evolution against time or tonnage. The equation contains an initial stage that corresponds to a rapid settlement of the ballast after tamping ($1 - e^{-\alpha.x}$) and a second phase of linear settlement against time ($\gamma + \beta.x$) that happens at a much lower rate. The typical shape is illustrated in Figure 11.

$$y = \gamma.(1 - e^{-\alpha.x}) + \beta.x \quad (5)$$

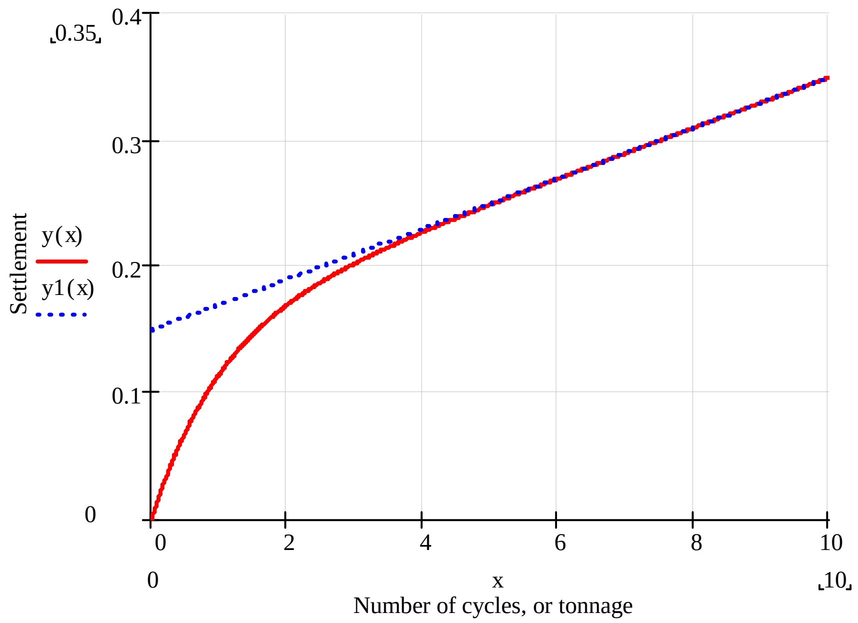

Figure 11: vertical settlement of ballasted track against tonnage or time

Some of these models such as the ones proposed by Sato [69] have the potential to make better use of the data generated from VTI models by including quantities such as:

- The ballast pressure (or the dynamic force passing through one sleeper divided by its area)
- The ballast acceleration (or the acceleration of the sleeper mass bouncing on the ballast)

The issue is that only VTI models with enhanced track models, i.e. modelling individual sleepers, may predict these quantities directly, and the standard moving track models found in MBS software do not usually output such values.

4.2.2 Limitations

The main issue with settlement models is that most of them are empirical in nature and the specific terms in the equation are usually ‘tuned’ to match data from specific track locations or countries. They are very difficult to transpose from one track condition to another, where traffic type, local geology, local maintenance regime and track design may vary significantly. They have therefore principally been used for comparative studies at one location with different rolling stock.

Another aspect is that they have been focused on ballasted track only. Similar equations for the case of alternative track constructions such as concrete/steel slab and floating slab track do not yet exist. Although in the presence of such construction the ballast layer is eliminated, it is likely that some sort of differential settlement of the subgrade may still be present especially in regions of transition between conventional ballasted track and slab track or wherever local subgrade weaknesses are present.

5. A classification of currently used high resolution VTI models

As part of deliverable 1.3.6, a list of vehicle-track interaction models used within INNOTRACK or relevant to the work carried out in INNOTRACK across work-packages SP2 (track), SP3 (S&C) and SP4 (rail) has been compiled.

The term high resolution (HR) has been used in this report to describe these models. It is employed here in the sense that:

- they are mechanistic models, i.e. producing engineering quantities derived from 1st principles of mechanics and dynamics,
- they include a representation of all three parts of the system: vehicle, track and wheel-rail contact,
- they include a level of details that corresponds to the current state-of-the-art in at least one of these three categories.

Table 1 contains the list of tools[5] split according to four categories:

- A] Vehicle focused tools based on Multibody System (MBS) dynamics
- B] Track focused tools based on Finite Element Method (FEM) and/or MBS.
- C] Wheel-rail contact focused tools mostly based on FEM or other method.
- D] 3rd party models or post-processing applications that can link to either of the above for further track damage analysis.

	Tool	**Used by / in SP#**	**Developer/Owner**	**Validation**
A] Vehicle models - Multibody System dynamics (MBS) tools				
A1	Vampire	MMU, NR / SP1	Delta Rail Group Ltd	Validation section in user manual (vehicle only). Participation in several benchmark exercises for vehicle and contact dynamics (annexe 7.2).
A2	Simpack	MMU, DB / SP3.1 (D3.1.5)	Intec GmbH	References to users validation available from website. Participation in several benchmark exercises for vehicle and contact dynamics. Demonstrators S&C site modelled in SP3.1 and compared with other MBS tools.
A3	VI-Rail (with FTSM)	MMU / SP2.3 & SP1 (D2.3.2, D2.3.5)	Vi-Grade GmbH	Participation in several benchmark exercises for vehicle and contact dynamics. Some laboratory validation for parts of FTSM. Comparison with detailed track FE model in SP2.3
A4	Gensys	Chalmers, Banverket / SP3.1 (D3.1.5)	DEsolver	Participation in several benchmark exercises for vehicle and contact dynamics.

[5] Any strategic models which were previously described in deliverable 1.3.2 are not included in Table 1.

	Tool	Used by / in SP#	Developer/Owner	Validation
A5	DIFF3D	Chalmers, Banverket / SP3.1 (D3.1.5)	Charmec	Numerous reference papers. Demonstrators S&C site modelled in SP3 and compared with other MBS tools.
B] Track models – FE and/or MBS tools				
B1	DIFF	Chalmers SP4.2 (D4.2.4, D4.2.5)	Charmec (Jens Nielsen)	Numerous reference papers. Field test measurement of wheel-rail contact force and rail bending moment. Applied to the prediction of rail bending moment generated by wheel flats.
B2	Track FE model	LCPC / SP2.2	LCPC	Some field tests in SP2. Applied to the study of lime cement column reinforcement.
B3	Sleeper on ballast box FE models (2D/3D)	T U Prague / SP2.1 and 2.2 (D2.1.3, D2.1.5, D2.1.16, D2.2.9)	Czech Technical University, Prague	Validated against laboratory tests in SP2.2 (see deliverable D2.1.3). Applied the study of geosynthetics.
B4	Multi-layer track FE models (2D/3D)	Banverket / SP2.2 (D2.2.5)	Banverket	Back analysis with site stiffness measurement. Applied to the study of lime cement column reinforcement.
B] Wheel-rail contact models – FE and/or MBS or other tools				
B5	Wheel-rail FE model with fatigue model	Chalmers / SP4.2 (D4.2.3)	Chalmers (Sandstrom & Ekberg)	Applied to the study of insulated joints degradation.
B6	Hybrid MB-FE model of wheel on rail/track	TU Delft / SP4.2 (D4.2.4)	TU Delft	Squat growth process validated against field data. Applied to the study of initiation and growth of squats.
B7	'dynarat' or 'brick' model	Newcastle U. / SP4.2 (D4.2.5)	Kapoor, Franklin and Fletcher	Validation against twin-disc experiment. Applied to the study of rail wear and crack initiation.
B8	Wheel-rail FE model	Chalmers / SP4.3 (D4.3.5)	Chalmers (Kabo)	Validation against twin disc test and full scale test from VAS and DB. Simulation of material deformation.
B9	BCCM (Bouncing Contact Conicity Modelisation)	VCSA / SP3.1 (D3.1.4)	VCSA	Comparison with other MBS contact codes. Applied to the analysis of S&C geometries and contact condition.
C] Other 3rd party/plugin applications or formulae that can link to the above				
C1	Tγ weighted function (included into WLRM)	Various / mostly SP1	Originally British Rail Research, further developed by TTCI and Network Rail.	Validation against site measurement and twin disk experiments.

	Tool	Used by / in SP#	Developer/Owner	Validation
C2	Shakedown	Various	K.L. Johnson	Experimental laboratory testing
C3	Wear models (Archard, sliding wear, etc.)	Various		Experimental laboratory testing for wear maps
C4	Empirical settlement formulae	MMU / SP2 (restricted to ballasted tracks)	Various: Sato, TU Berlin, UK, France…	Experimental testing on specific routes/locations

Table 1 : Table of vehicle-track interaction models

6. Conclusions

Deliverable 1.3.6 contains a review of the state of the art of vehicle track interaction models as a method for determining track degradation rates. References to existing reviews found in the literature are included in the annexe with a summary for all of them. Models found in the literature are split between the vehicle focused models based on multibody system (MBS) dynamics techniques and the track focused models now mostly based on Finite Element Method (FEM). Advantages and disadvantages of both types of models are explained such as the limitation of the track models included within current MBS software of the lack of representation for the vehicle dynamics and the wheel-rail contact kinematics in FE based track models.

In a second section the damage mechanisms that can be studied using the VTI models predictions are listed and described, e.g. wear and rolling contact fatigue using the energy transfer method, or ballast settlement calculations. Validation issues and future development are discussed throughout wherever applicable.

Finally a table is included with all the models that were used within INNOTRACK or that are relevant to the work carried out in INNOTRACK with some reference to which work-package and which deliverables they refer to.

In terms of validation and use of the VTI tools for LCC the following remarks may be drawn:

As mentioned in section 3.3 the VTI models presented in this report are mechanistic tools which primary function is to predict the dynamic behaviour of the vehicle and track expressed in the form of basic physical quantities such as forces and accelerations. In this respect the tools presented here are based on fully validated computer codes which are often used in wider application across other industries (e.g. MBS codes are widely accepted tools in the automotive industry, FE analysis software are also well used for wide range of applications). Railway specific MBS tools in particular have been benchmarked regarding the vehicle dynamic behaviour and the contact prediction (see Manchester Benchmark and contact benchmark in annexe 7.1).

The question of validation is therefore more relevant to the next part of the process which is how well are these physical quantities re-used in 3rd party damage models that express the degradation of track principally in terms of:

- material removed in µm for the rail profile, per travelled distance
- probability for RCF cracks to develop against traffic types and tonnage
- settlement in mm for the rail level or ballast layer, against time or tonnage.

This calculation stage highly relies on the availability and the quality of measured data in order to calibrate the damage models to get the best relationship between the fundamental forces/accelerations and the corresponding evolution of the rail profile or of the ballast level. A small change in the calibration parameters may end up with a big change in the deterioration results after several millions cycles. As seen in this report, several of the damage models have been calibrated against specific empirical data and the validation of one case study does not mean that the model can be re-applied directly to another case. A number of parameters will influence the results: traffic conditions (vehicle types, axle loads, speed…), local geology (influence of subgrade), weather conditions (on ballast properties for example), wheel and rail material, etc.

The use of VTI models together with degradation models is therefore mostly applicable to well targeted comparative studies for which the measured data used for validation can safely be re-applied to the evaluation of a small step change of the vehicle-track system. One important aspect of the current review is highlighting an existing gap in VTI tools for the simulation of innovative track systems that that have been presented in INNOTRACK SP2.3 for example. Very few VTI models are able to take into account other than ballasted track constructions and the current damage models are not able to make the difference between conventional ballasted tracks and these alternative innovations.

This review also reveals a gap between the vehicle dynamic models, which are standardised and largely accepted tools within the industry, and track models which tend to be used by universities for research purposes on a smaller and more scattered scale. The benchmarks from [44, 45] illustrate the wide range of track dynamics models developed by different people using different methods, and also the scatter of prediction results obtained, even for the a well defined investigation. Knothe and Grassie's comment in their 1993 review [1] still seem pertinent today as to how well track models can predict the behaviour of non-linear events along the track and how well they can predict the ballast behaviour and its degradation. It seems more effort should be spent both from the academia and from the infrastructure managers in helping validating these dynamic track models. Ideally, track models should reach the same maturity as that reached by vehicle dynamics software in the 1990s and a standardised modelling approach and validation methodology should be defined for future research to take full advantage of current software capabilities. In this respect, it is the role of the industry to provide the means for academia to achieve this maturity, mainly by access to accurate and substantial field validation data. Reciprocally it is the role of academic researchers to use these high resolution tools to gain understanding and transfer the knowledge back to the industry. This would take the form of standard reviews and guideline reports on maintenance and design practices. But more importantly, mechanistic rules should be implemented into lower resolution tools that have the potential to be more widely applied by field engineers. Such tools have been implemented and used in the UK for example with VTSIM, partly relying on high resolution vehicle dynamics modelling.

7. Annexes

7.1 Existing review on vehicle track interaction modelling

7.1.1 Vehicle-track interaction in the mid-high frequency range

Knothe K. and Grassie S.L., 'Modelling of railway track and vehicle/track interaction at high frequencies', *Vehicle System Dynamics*, Volume 22, issue 3 & 4 (1993), pages 209 – 262

This review is concerned with high frequencies vibration and noise (from 20 up to 5kHz, the upper limit of human hearing) as produced by vehicle/track interaction models. At lower frequencies, the dynamic behaviour of the track is not so significant. The upper limit was selected because the calculated wheelset and track response spectra then contain the most dominating components responsible for railway noise. Models for rail, sleeper and wheelset that are valid in the investigated frequency range were identified, however no solution was available at the time to predict the dynamic behaviour of rail pad and ballast, nor their long-term behaviour. This was regarded as the most promising area for future research.

Knothe and Grassie draw a list of how the most sophisticated vehicle and track model would be constructed. Based on all models reviewed it would include:

1. a vehicle comprising a body, bogies, primary and secondary suspension and an elastic wheelset;
2. wheel-rail contact for which full non-linear, non-steady-state analysis had to be undertaken both normally and tangentially;
3. a rail modelled as an infinite, discretely supported combination of individual plates or beams representing the head, the web and foot, with shear deformation and rotary inertia included;
4. rail pads modelled as spring in series with spring and dashpot in parallel;
5. sleepers represented as 3 dimensional bodies with varying cross sectional dimensions;
6. ballast represented as a layer with mass and with elements of stiffness and damping between the massive layer, the sleepers and the substrate;
7. a substrate represented as a three dimensional half space;
8. irregularities in the track support, both from spacing of the sleepers and from there being voids under the sleepers and missing rail pads.

Of course the authors then recommend that models be simplified as much as possible according to the type of application and results investigated, particularly in view of carrying out parameter studies for which fast models are required (for example assuming continuous support condition and frequency domain analysis). On the other hand finite element models and time stepping integration methods are required for the analysis of non-linear events in the system such as uneven sleeper spacing, missing rail pads, voided sleepers or non-linearities in the contact (loss of contact for example).

According to Knothe and Grassie the main challenge for future vehicle-track models in the high frequency range is their ability to represent accurately the wide range of railpads and their non-linearities, as well as the ballast dynamic behaviour. Enhanced validated models should be able to predict the damage made to these track components and to the ballast layer, so that they can help reduced maintenance cost of the infrastructure. Particularly, models should be able to handle non-linearities as perfect homogenous track do not exist in reality.

Their final conclusion is that models should be developed and solution procedure adopted which are appropriate to the problem of interest and as simple to use as is reasonably possible. Models should also always be tested satisfactorily by comparison of the predicted behaviour with that observed.

Popp K., Kruse H. and Kaiser I., 'Vehicle-track dynamics in the mid-frequency range', *Vehicle System Dynamics*, Volume 31, Issue 5 & 6 June (1999), pages 423 - 464

This review particularly focuses on the mid-frequency range defined above 50Hz (below which, one is mainly concerned with passenger comfort and stability) and below 500Hz (above which acoustical problems occur). These frequencies are considered by the authors left out from previous researches.

The authors identify an existing split between vehicle models (with simplified track inputs) and track models (with simplified vehicle loading), still relevant today as will be seen later in this report.

For the vehicle models, elastic wheelset and coupling of the wheelset through the bogie should be included. This is discussed in the view of vibration and damage analysis of the wheels however it should be also relevant to the damage made to the rails. The authors argue that wear models for the wheel based on frictional power hypothesis are not sufficient and should be improved in the future, i.e. should account for plastic deformation, and *other mechanism based on cracks or material non homogeneities. This comment is also valid for the case of the rail damage.*

Regarding the track models, they should include detailed modelling of the rail flexibility, the pads, the ballast and subgrade. Only bending of the rail is important and cross section deformation can be neglected. Shear behaviour is important above 500Hz, notably torsional effect. Models have been categorised as frequency domain or time domain models. The first ones are faster and provide a more global response of the system across a wide frequency spectrum, however they are linear models and cannot fully take into account the complex properties of pads and ballast. Time domain models are usually more complex, i.e. they may include non-linear features/properties, which also make them much slower. The ballast physical behaviour is seen as the most problematic to model due to the granular nature of this layer, which stiffness highly depends on the void ratio (different under sleepers and in between sleepers), the loading velocity and the stress state. The damping properties, mostly due to dry friction, are unknown.

The authors comments on the fact that simulating the track behaviour is still a heavy task in terms of calculation power and time. Therefore the number of publication dealing with the simulation of the complete vehicle-track interaction in the mid-frequency range was at the time very rare.

7.1.2 Multibody system dynamics techniques

Shabana A. A. and Sany J. R., ‘A survey of rail vehicle track simulation and flexible multibody dynamics’, *Nonlinear Dynamics*, Volume 26, Number 2, October (2001), pages 179-212

This survey describes the techniques of multibody dynamics and how it is applied to the study of rail vehicle and track simulation. Since deliverable 1.3.6 is reviewing a significant number of rail specific multibody dynamic software, the above paper may be useful in gaining a better understanding of the technology employed as well as the challenges these software face, namely the addition of flexible elements such as the rail or wheelset axle that can be embedded together with a detailed wheel-rail contact calculation routine.

7.1.3 Wheel-rail contact damage mechanisms and track deterioration

Nielsen J.C.O., Lunden R., Johansson A., and Vernersson T., ‘Train-track interaction and mechanisms of irregular wear on wheel and rail surfaces’, *Vehicle System Dynamics*, Volume 40, issue 1-3, September (2003), pages 3-54

Their review is particularly focused on the aspect of irregular wear of the rail and wheel surface such as short pitch rail corrugation (2.5 to 8cm with 10µm amplitude), wheel corrugation (5 to 7cm with 10µm amplitude) and wheel polygonalisation (1-5 harmonics around wheel circumference with around 1mm amplitude).

The authors also make the same distinction between models solved in time domain and frequency domain, mentioning the advantages and disadvantages of both types.

1) *Frequency based models are usually of the ‘moving irregularity’ type which can be thought of as an imaginary strip containing the wheel/rail irregularity that is pulled at a steady speed between the vehicle and the track models. The system has to be completely linear (no transient dynamics is considered). Frequency response functions (FRF) of the different parts of the system are coupled to form the appropriate transfer function between input and outputs.*
2) *Time domain based models are of the ‘moving mass’ types with the vehicle model travelling along the track at speed. Time stepping integration techniques are used to solve the system and non-repetitive properties may be included such as: variation in sleeper distance, voided sleepers, state dependent rail pads, scattered ballast/subgrade properties but also non-linear contact mechanics, loss/recovery of contact, etc. Advancement in computer power also allow detailed flexible track models to be developed based on Finite Element method (FEM). Modal superposition method is also very attractive techniques for the analysis of linear time-invariant components, e.g. rail, with a reduced number of equations and a limited loss of accuracy.*

Regarding the vehicle models, the authors make the distinction between low frequencies (below 20Hz) for which typical Multibody Dynamics (MBD) software are generally used, and the higher frequencies for which wheelset flexible modes are important as well as the coupling of two wheelset in a bogie through the rails (and the resulting wave reflection between the wheels). These aspects are not negligible for damage studies on the wheel and the rail, e.g. short-pitch corrugation.

The behaviour of track models in the vertical and in the lateral direction are explained with the help of typical receptance graphs based on the work from Ripke and Knothe [46] showing for example the rails and sleepers vibrating in phase on the ballast at around 100Hz, or the rails vibrating out of phase with the sleepers at around

400Hz. The pinned-pinned frequency of the rail is found at around 1000Hz. In the lateral direction similar behaviour is observed, however with lower resonance frequencies. At high frequencies the cross sectional deformation of the rail becomes important so that rail head and rail foot need to be modelled as two beams interconnected by the web. Rail pad stiffness is identified as having a strong influence on the high frequency dynamic behaviour of the track. They are also highly non-linear depending on pre-load and excitation frequency; however these non-linearities are mostly influential for noise and vibration problems, rather than damage and wear. The sleepers are straight forward to model either as rigid bodies or flexible beams. The authors comment on the principal unresolved question regarding the modelling of the ballast and subgrade, with a strong dependence on preload as well as a high spatial stiffness variation (different reaction forces from one sleeper to the next) and a high number of hanging sleepers.

This review also includes an informative section on damage mechanisms responsible for wheel and rail corrugation: wear, plastic deformation and rolling contact fatigue. However they comment that in most models, only wear is accounted for. Several empirical wear indices are mentioned such as Archard [56] volume of material removed or the Tγ function based on the frictional power [70]. These will be described later in the present report. The long term simulation of wear is usually done using a combination of vehicle-track interaction model (short terms dynamics loads and condition) and damage model (long term wear process) in a feed-back loop. This will also be described later in this report.

Dahlberg T., 'Some railroad settlement models – a critical review', *Journal of Rail and Rapid Transit*, IMechE Part F, Volume 215 (2001)

This review mainly concentrates on the issue of railway track and ballast settlement and the models used to predict it. The Author summarise the review by saying that: "there do not seem to be a generally accepted damage and settlement equations describing the long-term behaviour of the track" in the sense of its global behaviour as measured at rail level and he also extend this conclusion to the particular case of the ballast material. Most models found in the literature are empirical and mostly base their prediction on the number of loading cycles and/or the magnitude of the loading. The major flaw of existing models resides in their failure to include the physical properties of the ballast and sub-ground materials.

Factors that influence the design and the maintenance of tracks are partly known: history of maintenance, environmental conditions and traffic, but the influential factor of ballast and subgrade characteristics is in most cases non-existent. Moreover, regarding maintenance of existing track, the subgrade is not part of any maintenance regime and it is often the cause of poor track quality.

Track settle as a results of permanent deformation in the ballast and underlying soil caused by repeated traffic, and it occurs in two phases:

1) *An initial fast settlement directly after tamping due to the consolidation of the ballast layer (closing of the gap between ballast stones).*

2) *A second quasi-linear phase that happens slowly over time (or loading) due to several mechanisms of densification (volume reduction because of particle rearrangement, particle fracture, abrasive wear and sinking of the ballast into sub-ballast and subgrade) and of inelastic behaviour (micros-slip between ballast stones and ballast migration away from the sleeper) of the ballast and subgrade.*

This can be described mathematically by a function of the type: $y = \gamma(1 - e^{-\alpha x}) + \beta x$ *where* x *represent the loading of the track and* $\gamma + \beta x$ *provides the long-term linear settlement and the factor* $(1 - e^{-\alpha x})$ *provides the initial stage of rapid settlement. Some of the more advanced studies included further input quantities to 'tune' the equations parameters, for example Sato [69] uses sleeper to ballast pressure, sleeper acceleration and the square of the loading velocity to obtain* β *. Sato also used another version based on the ballast pressure with two possible scenarios: (a) the pressure is below a certain value and no settlement occurs (ballast reacts elastically) or (b) beyond a certain threshold of pressure settlement function is applied. Other models are generally more simplistic and mostly take into account the loading on the track. The Sato models will be further described in this report for the interest it can provide with regards to its use in combination with vehicle-track interaction models.*

Dahlberg T., 'Railway track dynamics – a survey' (2003) and 'Railway track settlement – a literature review' (2003) from SUPERTRACK (Sustained Performance of Railway Tracks) European project.

Tore Dahlberg's review start by listing all the components making up a typical ballasted track, and sum up the main functions of the track which are to guide the train and carry the load. Dahlberg describes the overall dynamics properties of the track by discussing the use of receptance graphs obtained from harmonic excitation using hydraulic actuators (up to 200Hz) or impact load methods such as a sledge hammer (higher frequencies) on real track. The receptance graph provides the peaks of resonance of the track in the vertical directions across a wide frequency range. It shows the lightly damped (narrow peak) pinned-pinned resonance and anti-resonance *peaks measured in between sleepers and above sleepers respectively. Other modes highly damped by the ballast are the rails bouncing on the sleepers (between 200 and 600Hz) and the rails and sleepers bouncing together on the ballast (50 to 300Hz). Dahlberg discusses the potential presence on low bearing soil stiffness of a well damped peak of resonance in the range 20 to 40Hz. This is due to the interface between ballast and subgrade and if a numerical model needs to take this into account, then an additional layer needs to be modelled for the ballast mass and the subgrade stiffness. Often the ballast masses are also connected to adjacent ones by a spring and damper element to represent the deflection of the subgrade in the longitudinal direction (along the track). Dahlberg mentions the fact that Euler-Bernouilli beam approximation for the rail are limited above a certain frequency at which the characteristic wavelength becomes comparable to the height of the rail, in such case the Rayleigh-Timoshenko theory that takes into account shear deformation and rotary inertia, needs to be applied. The non-linearity of the track response as a function of loading and also as a function of the position along the track is highlighted. Before reviewing existing analytical and numerical models, Dahlberg discusses some of the main sources of train-track excitations:*

1) *Rail head corrugation or short wavelength irregularity (roughly 30 to 300mm) has typically been classified according to wavelength by various authors. Their consequence is principally to induce vibration into the track due to the disproportion between the wheel inertia and the rail's. At certain speeds resonance may be achieved with the sleeper, most likely leading to damage to rail pads and fastenings, ballast degradation and track settlement. Many theories have been disputed by many authors and there are most likely a number of reasons for the appearance of short wavelength rail irregularity, and the authors refers to the review by Sato [71] for further information.*
2) *Long wavelength irregularities (300mm or longer) which include out-of-roundness wheel but also the cyclic variation of track stiffness due to sleeper passing frequency, rail manufacturing process, variable stiffness at switches and crossings and embankment settlement.*
3) *Impact loads due to wheel flats, rail joints and switches.*

Vehicle track interaction models are described starting from the simplest analytical representation of a rail beam on continuous elastic foundation (Winkler) or a moving mass on a simply supported beam for vehicle-bridge interaction, through to numerical models of beam on discrete support made of one or several layers, up to complex 3D Finite Element models of the rails, sleepers, ballast and subgrade. The same classification of the solution technique into frequency and time domain is drawn with the same advantages and disadvantages mentioned by other authors as mentioned above in this table.

Useful information about the dynamic properties of the track individual components is also found in this review to better understand how each components or layer needs to be modelled.

Popp K., Knothe K. and Popper C., 'System dynamics and long-term behaviour of railway vehicles, track and subgrade: report on the DFG priority programme in Germany and subsequent research', *Vehicle System Dynamics*, Volume 43, Issues 6-7, pages 485-521 (2005).

This paper provides a broad and detailed picture of the state-of-the-art research topics in Germany from the late 1990's up to 2005 under the umbrella of the Deutsche Forschungsgemeinschaft (DFG) Priority Programme on 'system dynamics and long-term behaviour of vehicle. Track and subgrade'. Early development of numerical models is presented against the need in the post-war era to achieve high-speed travel and therefore determine the physical limits of the steel wheel on steel rail by means of vehicle stability and critical speed analysis. This is in the late 70's, early 80's that vehicle multibody codes emerge in Germany and other countries to forms the basis of currently well established software such as Medyna or Simpack. In the late 80's and early 90's, new problems emerge (irregular ballast settlement and ballast deterioration, short wavelength rail corrugation, out-of-round wheel and noise) together with the expansion of high-speed traffic across Germany and basic demand for new development in modelling tools also increases, fulfilling the following requirements:

- *Vehicle dynamics had to be substituted by vehicle-track dynamics, i.e. by real system dynamics*
- *The restriction to low frequency range had to be abandoned. In addition, the medium-frequency range had to be included in the dynamic investigations*
- *The analysis of the short-time system dynamics had to be augmented by the analysis of the long-term behaviour of different vehicle and track components.*

The main outcome from the various DFG Priority Programme sub-projects are:

- *Linear and nonlinear simulation of vehicle/track interaction in the medium frequency range has been well understood*
- *Simulation of ballast and subgrade as an infinite, continuous or layered half-space with or without inclusions has also been well understood*

- *System dynamics of wheelset and bogie in the medium frequency range could not cover all problems however it was demonstrated that the tools required are fully available*

Concerning the long-term behaviour the conclusions were not as clear:

- *Two hypothesis for out-of-round wheels were investigated and explain qualitatively, however research across Europe needs to be brought together to summarise different aspects of this problem*
- *A big step forward was made in the understanding of settlement and deterioration of ballast and subgrade phenomena. Conclusions are that within 5 to 10 years ballast settlement will be practically understood*

Future problems to be solved with the necessary support of railway infrastructure managers, particularly for the experimental validation of numerical models, are discussed as follow:

- *In situ measurement techniques for the wheel-rail contact patch*
- *Damage phenomena of the running surface of wheel and rail: material behaviour for extreme loading conditions, long-term behaviour, etc.*
- *Fundamental aspects of friction and wear in rolling contact*
- *Condition of rail surfaces either naturally or artificially*

It has to be noted that a lot of research has been happening since this review was published, particularly on the initiation of rolling contact fatigue (which was not part of the DFG Priority Programme subject at the time) particularly in the UK, and on the use of grinding campaigns to controls it.

The second part of the paper describes several of the models used for both vehicle and track categories, similar to the ones already described in [2]. Other interesting research activities not mentioned thus far are the FE modelling of the wheel and rail contact highlighting some limitations of the Hertzian methods, and also the use of Molecular Dynamics (MD) method for simulating the dynamic and long-term behaviour of ballast.

Enblom R., 'Deterioration Mechanisms in the wheel-rail interface with focus on wear prediction: a literature review', Vehicle System Dynamics, Volume 47, Number 6, pages 661-700, June (2009)

The review from Enblom focuses on the deterioration mechanisms at the wheel-rail interface, which is one of the main maintenance cost driver in the industry, and also a complex subject involving practical as well as theoretical cross-disciplinary expertise. Several mechanisms are involved: abrasive and adhesive wear, plastic deformation, rolling contact fatigue requiring different fields of expertise: e.g. tribology, solid mechanics, vehicle dynamics… Analysing the evolution of a wheel or rail profiles thus require the simulation of the traffic and environment condition, this is generally done with a multibody system (MBS) dynamics analysis (works out the dynamic behaviour of the vehicle on the track), which also includes a contact mechanics programme (can work out the contact stress, creep forces, takes into account the friction coefficient etc.). MBS tools work in the milliseconds resolution, and one of the technicalities is to link this to wear models that predict the evolution of the profiles shape typically in month resolution. Surface plasticity may be included but so far this is most often neglected. The above type of analysis traditionally relies on the energy dissipation in the contact to determine the amount of material loss. Several techniques are used to link this energy quantity to a material loss quantity such as weighted functions – for Tγ based functions - or wear maps (based on normal contact stress, tangential stress or slip and lubrication conditions and populated from laboratory testing) – for Archard based functions.

The future of wheel-rail deterioration mechanism prediction relies on the further development of integrated analysis tools that can take into account several mechanism at the same time.

7.2 Benchmarking of vehicle-track interaction models

7.2.1 General VTI model (high frequencies)

Grassie S.L., 'Models of Railway track and train-track interaction at high frequencies: Results of benchmark test', *Vehicle System Dynamics*, Volume 25, issue S1 (1996), pages 243 - 262

Abstract: *Results have been compared of eight contributions to a benchmark test which was written for programs developed to examine the high frequency dynamic interaction of railway vehicles and the track. Participants were requested to consider a vehicle passing over uniform, sinusoidal corrugation, and to calculate the vertical rail acceleration and various forces and bending moments in rail and sleeper. From the results obtained from a wide variety of time and frequency based solution techniques, it is concluded that substantially identical results can be obtained from both types of model in the majority of conditions considered in the test. There is a greater variation* *in results between the 5 frequency domain models used in these submissions than between the 2 high frequency time domain models.*

7.2.2 Multibody Dynamics software

Iwnicki S.D., 'The Manchester benchmarks for rail simulators – an introduction', *Vehicle System Dynamics*, Volume 29, Issue S1, pages 717-722 (1998)

Abstract: *Two simple vehicles and four matching track cases are presented to allow comparison of the capabilities of the various computer simulation packages currently being used to model the dynamic behaviour of railway vehicles. The benchmarks presented here were agreed at the International Workshop on COMPUTER SIMULATION OF RAIL VEHICLE DYNAMICS at Manchester Metropolitan University on June 23rd and 24th 1997.*

Iwnicki S.D., ' The Manchester Benchmarks for Rail Vehicle Simulation', *Vehicle System Dynamics*, Volume 31 Supplement, Taylor & Francis, 1st edition January 1 (1999) – Hardcover, **ISBN-13:** 978-9026515514

Abstract: *This book contains the results of the Manchester Benchmarking exercise for railway vehicle dynamics simulation packages. Five of the main computer packages - Adamsrail/Medyna, Gensys, Nucars, Simpack, Vampire - currently used for this purpose have taken part in the exercise. The results are presented in the form of tables and plots comparing how each package predicts the vehicle behaviour. These results are discussed and the differences analysed. Comments made by simulators themselves are set out in a separate section. In addition to the simulation results, each simulator has supplied a statement of methods. This statement sets out the way in which each package was used to carry out the simulations and details the approximations made. In addition, six further papers give examples of the way railway vehicle dynamic behaviour is simulated in a variety of applications using different packages.*

7.2.3 Railway Track Dynamics tools

Steffens D. and Murray M., 'Establishing meaningful results from models of railway track dynamic behaviour', In 8th International Heavy Haul Conference, 14-16 June, Rio de Janeiro, Brazil (2005)

Abstract: *Traditional empirical methods of designing railway track rely on simplistic impact factor methods that crudely represent the complex dynamic behaviour of track and train interaction and of the defects that give rise to damaging forces. Various analytical models have been developed around the world to help the track design engineer better understand the consequences of variations and innovations in track design. The creators of six recently developed models from Canada, China, Germany, Sweden, USA and UK were invited to participate in a benchmarking exercise to allow comparison of the operation, outputs and applicability of those models. Although detailed instructions were given to the benchmark participants, variations in interpretation, complexity and underlying theory of each model led to differences in outputs. This paper provides some guidance in interpreting these differences and compares the results to those obtained from traditional design processes.*

Leong J., Murray M. and Steffens D., 'Examination of railway track dynamic models capabilities against measured field data', In: International Heavy Haul Conference Specialist Technical Session, High Tech in Heavy Haul, June 11-13, Kiruna, Sweden. (2007)

Abstract: *The performance of railway track under dynamic loading is extremely complex due to the interdependent and sometimes non-linear behaviours of track components. Many and varied computer models of train-track dynamics have been developed around the world to try and analyse these behaviours. To compare, test and validate the capabilities and outputs of six available computer models, an evaluation exercise was undertaken to benchmark the participating models against each other and against measured field data collected at a track test site in Victoria, Australia. The outcomes of this paper will assist railway engineers in selecting an appropriate model for their requirements.*

8. References

1. Knothe, K. and S.L. Grassie, *Modelling of Railway Track and Vehicle/Track Interaction at High Frequencies.* Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, (1993). 22(3): p. 209.
2. Popp, K., H. Kruse, and I. Kaiser, *Vehicle-Track Dynamics in the Mid-Frequency Range.* Vehicle System Dynamics, (1999). 31: p. 423-464.
3. Shabana, A.A. and J.R. Sany, *A Survey of Rail Vehicle Track Simulations and Flexible Multibody Dynamics.* Nonlinear Dynamics, (2001). 26(2): p. 179-212.
4. Dahlberg, T., *Some railroad settlement models—a critical review.* Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, (2001). 215(4): p. 289.
5. Dahlberg, T., *Railway track settlements - a literature review*. Division of Solid Mechanics, IKP, Linköping University, Linköping, Sweden, 13-01-2003 (2003)
6. Dahlberg, T., *Railway track dynamics - a survey*. Division of Solid Mechanics, IKP, Linköping University, Linköping, Sweden, (2003)
7. Enblom, R., *Deterioration mechanisms in the wheel-rail interface with focus on wear prediction: a literature review.* Vehicle System Dynamics, (2009). 47(6): p. 661-700.
8. Nielsen, J.C.O., R. Lunden, A. Johansson, and T. Vernersson, *Train-Track Interaction and Mechanisms of Irregular Wear on Wheel and Rail Surfaces.* Vehicle System Dynamics, (2003). 40(1-3): p. 3-54.
9. Popp, K., K. Knothe, and C. Pöpper, *System dynamics and long-term behaviour of railway vehicles, track and subgrade: report on the DFG Priority Programme in Germany and subsequent research.* Vehicle System Dynamics, (2005). 43(6-7): p. 485-521.
10. Iwnicki, S., ed. *The Manchester Benchmarks for Rail Vehicle Simulation*. Vehicle System Dynamics, ed. S. Iwnicki. Vol. 31 (supplement). (1999), Swets & Zeitlinger. ISSN 0042-3114.
11. Garg, V.K. and R.V. Dukkipati, *Dynamics of railway vehicle systems*. (1984): Academic Press Canada. 407.
12. Iwnicki, S., *Handbook of Railway Vehicle Dynamics*, ed. S. Iwnicki. (2006): CRC Press. 776.
13. Shabana, A.A., *Dynamics of Multibody Systems*. 2nd Edition ed. (1998): Cambridge University Press. 372.
14. Knothe, K., R. Wille, and B.W. Zastrau, *Advanced Contact Mechanics - Road and Rail.* Vehicle System Dynamics, (2001). 35(4-5): p. 361-407.
15. Evans, J. and S.D. Iwnicki. *Vehicle dynamics and the wheel/rail interface*. in Proc. I.Mech.E. Seminar 'Wheels on Rails – An update' April (2002), London, UK,
16. Iwnicki, S., *Simulation of wheel-rail contact forces.* Fatigue & Fracture of Engineering Materials and Structures, (2003). 26(10): p. 887-900.
17. Iwnicki, S., *Simulation Tools for Railway Vehicle/Track-Systems.* ZEVrail - Zeitschrift fur das gesamte System Bahn, (2004). 128. Jahrgang(4): p. 140-149.
18. Wallrapp, O., *Review of the Past Developments in Multibody System Dynamics at DLR - From FADYNA to SIMPACK.* Vehicle System Dynamics, (2004). 41(5): p. 339-348.
19. Frohling, R.D., *Wheel/rail interface management in heavy haul railway operations - applying science and technology.* Vehicle System Dynamics, (2007). 45(7): p. 649-677.
20. Knothe, K. and S. Grassie, *Modelling of railway track and vehicle-track interaction at high frequencies.* Vehicle System Dynamics, (1993). 22(3-4): p. 209-262.
21. Zacher, M. and F. Ambrogi. *Dynamics of a train over a flexible bridge*. in 15th European Mechanical Dynamics User Conference 15th Nov 2000 (2000), Roma, Italy,

22. Bezin, Y., S. Iwnicki, and M. Cavalletti. *Understanding the Wheel-Rail Interface using an Integrated Flexible Track Simulation Tool*. in MSC.Software 2005 Virtual Product Development Conference Europe, Middle East, Africa October 24 - 26 (2005), Munich, Germany,

23. Bezin, Y. *Development of a Flexible Track Model and its application to Advance Railway Track Design*. in 1st VI-Grade User Conference Nov. 13-14th (2007), Marburg, Germany,

24. Dietz, S., G. Hippman, and G. Schupp, *Interaction of Vehicle and Flexible Tracks by Co-Simulation of Multibody Vehicle Systems and Finite Element Track Models.* Vehicle System Dynamics, (2001).

25. Elkins, J., B. Brickle, N. Wilson, S. Singh, and H. Wu, *Track Structure Modeling with Nucars and its Validation.* Vehicle System Dynamics, (2002). 37 (supplement): p. 420-431.

26. Shu, X., N. Wilson, and C. Sasaoka. *Development of a real-time wheel/rail contact model in NUCARS and application to diamond crossing and turnout design simulations*. in 19th IAVSD Symposium 2005 August (2005), Milan, Italy,

27. Johnson, K.L., *Contact Mechanics*. (1985), Cambridge: Cambridge University Press. 452.

28. Pascal, J.-P. and G. Sauvage. *New method for reducing the multicontact wheel/rail problem to one equivalent rigid contact patch*. in Proc. 9th IAVSD Symp. (1985), Linkoping, Sweden: Swets & Zeitlinger, pp264-277

29. Kik, W. and J. Piotrowski. *A fast, approximate method to calculate normal load at contact between wheel and rail and creep forces during rolling*. in 2nd mini conf. on contact mechanics and wear of wheel/rail systems (1996), TU Budapest,

30. Ayasse, J.-B., H. Chollet, and J.-L. Maupu, *Paramètres caractéristiques du contact roue-rail*. INRETS (LTN), 225 January 2000 (2000)

31. Carter, F.W. *The electric locomotive*. in Proc. Institute of Civil Engineers. (1916), 221, pp221-252

32. Vermeulen, P.J. and K.L. Johnson, *Contact of non-spherical elastic bodies transmitting tangential forces.* J. of Appl. Mech. Trans., (1964). 86: p. 338-340.

33. Kalker, J.J., *Wheel-rail rolling contact theory.* Wear, (1991). 144(1-2): p. 243-261.

34. Kalker, J.J., *A Fast Algorithm for the Simplified Theory of Rolling Contact.* Vehicle System Dynamics, (1982). 11: p. 1-13.

35. Shackleton, P. and S. Iwnicki, *Comparison of wheel-rail contact codes for railway vehicle simulation: an introduction to the Manchester Contact Benchmark and initial results.* Vehicle System Dynamics, (2008). 46(1): p. 129 - 149.

36. Kik, W., D. Moelle, C. Bogo, and G. Ferrarotti, *The Manchester Benchmarks ADAMS/Rail - MEDYNA Statement of methods.* Vehicle System Dynamics, (1999). 31 (supplement): p. 49-65.

37. Telliskivi, T., U. Olofsson, U. Sellgren, and P. Kruse. *A tool and a method for FE analysis of wheel and rail interaction*. in 10th international ANSYS conference and exhibition 28-30 August (2000), Pittsburg,

38. Wiest, M., E. Kassa, W. Daves, J.C.O. Nielsen, and H. Ossberger, *Assessment of methods for calculating contact pressure in wheel-rail/switch contact.* Wear, (2008)(In Press).

39. Gurule, S. and N. Wilson, *Simulation of wheel/rail interaction in turnouts and special track work.* Vehicle System Dynamics, (1999). 33 (Supplement): p. 143-154.

40. Sebes, M., J.-B. Ayasse, H. Chollet, P. Pouligny, and B. Pirat. *Application of a semi Hertzian method to the simulation of vehicles in high speed switches*. in 19th IAVSD Symposium 2005 August (2005), Milan, Italy,

41. Vermeij, I. *Virtual Switch Testing Facility for Lightrail Vehicles*. in VPD Conference EMEA 2005 October 24-26 (2005), Munich, Germany: MSC Software,

42. Timoshenko, S. *Method of analysis of statical and dynamical stresses in rail*. in Proc. of 2nd International Congress for Applied Mechanics 12-17 September (1926), Zurich, Zwitzerland, pp407-418

43. Frýba, L., *Impacts of two-axle system traversing a beam.* International Journal of Solids and Structures, (1968). 4(11): p. 1107.

44. Leong, J., M. Murray, and D. Steffens. *Examination of railway track dynamic models capabilities against measured field data*. in International Heavy Haul Conference Specialist Technical Session, High Tech in Heavy Haul June 11-13 (2007), Kiruna, Sweden,

45. Steffens, D. and M. Murray. *Establishing meaningful results from models of railway track dynamic behaviour*. in 8th International Heavy Haul Conference 14-16 June (2005), Rio de Janeiro, Brazil,

46. Ripke, B. and K. Knothe, *Die unendlich lange Schiene auf diskreten Schwellen bei harmonischer Einzellasterregung.* VDI Fortschritt-Berichte, (1991). 155(11).

47. Lundqvist, A. and T. Dahlberg, *Dynamic train/track interaction including model for track settlement evolvement.* Vehicle System Dynamics, (2004). 41 (supplement): p. 667-676.

48. Lundqvist, A. and T. Dahlberg, *Load impact on railway track due to unsupported sleepers.* Proc. IMechE, Part F: J. Rail and Rapid Transit, (2005). 219: p. 67-77.

49. Lane, H., *Computational railway dynamics - Integrated track-train-subgrade modeling and simulation*. from Chalmers University of Technology, Department of Applied Mechanics, Goteborg, Sweden (2007)

50. Kabo, E., J.C.O. Nielsen, and A. Ekberg, *Prediction of dynamic train-track interaction and subsequent material deterioration in the presence of insulated rail joints.* Vehicle System Dynamics, (2006). 44(Supplement): p. 718 - 729.

51. Jaiswal, J., S. Blair, A. Stevens, S. Iwnicki, and Y. Bezin. *A system approach to evaluating rail life*. in Railway Engineering 2002 July 2002 (2002), London,

52. Lewis, R., A. Ekberg, S. Bruni, and R. Dwyer-Joyce. *A design tool for railway wheel incorporating damage models and dynamic simulations*. in Proceedings of the ASME Rail Conference 2005 March 16-18th (2005), pp pp 73-81

53. McEwen, I.J. and R.F. Harvey, *Interpretation of wheel/rail wear numbers*. British Rail Research, Derby, UK, TM VDY 004 July (1986)

54. Harvey, R. and I. McEwen, *The relationship between wear number and wheel/rail wear in the laboratory and the field*. British Rail Research, TM VDY 001 June (1986)

55. Burstow, M.C., A.S. Watson, and M. Beagles, *Simulation of Rail Wear and Rolling Contact Fatigue using the Whole Life Rail Model*, in *Railway Engineering*. (2003): The Commonwealth Institute - London.

56. Archard, J.F., *Contact and rubbing of flat surfaces.* Journal of Applied Physics, (1953). 24: p. 981-988.

57. Jendel, T., *Prediction of wheel profile wear--comparisons with field measurements.* Wear, (2002). 253(1-2): p. 89.

58. Jendel, T. and M. Berg, *Prediction of wheel profile wear.* Vehicle System Dynamics, (2002). 37 (Supplement): p. 502-513.

59. Bevan, A., P.D. Allen, and S. Iwnicki, *Development of an anti-RCF wheel profile*. Rail Standards and Safety Board, 110/10 August (2005)

60. Liu, Y., L. Liu, and S. Mahadevan, *Analysis of subsurface crack propagation under rolling contact loading in railroad wheels using FEM.* Engineering Fracture Mechanics, (2007). 74(17): p. 2659-2674.

61. Franklin, F.J. and A.J. Kapoor, *Modelling wear and crack initiation in rails.* Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, (2007). 221(1): p. 23-33.

62. Foletti, S. and H. Desimone, *Application of shakedown maps under variable loads.* Engineering Fracture Mechanics, (2007). 74: p. 527-538.

63. Johnson, K. and J. Jefferis. *Plastic flow and residual stresses in rolling and sliding contact*. in Proc Inst Mech Eng Symp on Fatigue in Rolling Contact (1963), London, pp54-65

64. Ponter, A., A. Hearle, and K. Johnson, *Application of the kinematical shakedown theorem to rolling and sliding point contacts.* Journal of the Mechanics and Physics of Solids, (1985). 33(4): p. 339-62.

65. Ekberg, A., E. Kabo, and H. Andersson, *An engineering model for prediction of rolling contact fatigue of railway wheels.* Fatigue & Fracture of Engineering Materials and Structures, (2002). 25(10): p. 899-909.

66. Burstow, M.C., *Whole Life Rail Model application and development of RSSB - Continued development of an RCF damage parameter*. Rail Safety and Standards Board, London, (2004)

67. Tunna, J., J. Sinclair, and J. Perez, *A review of wheel wear and rolling contact fatigue.* Proc. IMechE Part-F: Journal of Rail and Rapid Transit, (2007). 221: p. 271.

68. Ringsberg, J.W. and A. Bergkvist, *On Propagation of short rolling contact fatigue cracks.* Fatigue & Fracture of Engineering Materials & Structures, (2003). 26: p. 969-983.

69. Sato, Y., *Japanese Studies on Deterioration of Ballasted Track.* Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, (1995). 24(1 supp 1): p. 197.

70. Elkins, J.A. and B.M. Eickhoff. *Advances in non-linear wheel-rail force prediction methods and their validation*. in Proceeding of the ASME Winter Annual Meeting (1979),

71. Sato, Y., A. Matsumoto, and K. Knothe, *Review on rail corrugation studies.* Wear, (2002). 253(1-2): p. 130.