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INNOTRACK

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Glossary

The following terms used in this report should be interpreted as follows:-

Abbreviation / acronym	**Description**
Probability	always refers to probability in a statistical sense
Risk	The product of the probability of an event and the cost of the consequences of its occurrence
s.i.f	Stress Intensity factor
SFT	Stress free temperature
CWR	Continuously welded rail

1. Executive Summary

This report sets out a scientific approach to the determination of 'minimum action' timescales i.e. the timeframe within which the engineer must take action when a defect has been discovered.

For the purpose of this report, 'Minimum Actions' are defined as 'the actions that the engineer with responsibility for the safety of the line must take in the event that a defective or broken rail is discovered'. Thus the report is not about preventative maintenance strategies such as rail grinding. It is about the actions required and the timescale for action when preventative measures have failed to stop the development of a crack that threatens the integrity of the rail.

Within Europe, practices, in particular minimum action timescales, vary widely. This is not surprising since so do inspection regimes, and the two are closely connected. The longer the period between inspections, the greater the time available for defect growth and hence the shorter the time available (on average) to take action when a defect is discovered. Axle loads and traffic density would also be expected to have a major effect.

As a generality, it is not possible to predict, with confidence, the residual life of a specific rail containing a crack. There is too much scatter in material properties, support conditions, vehicle loadings, residual stresses etc. to enable this to be done. However a considerable amount of information is available on the range of properties, support conditions etc. that are experienced. Using these data and the process of Monte Carlo simulation, it is possible to predict, on a system wide or track category basis, what the effect of changes in minimum action time scales, inspection method or inspection period, traffic pattern or track construction will be on the proportion of defects that will result in rail breakage.

This is the essence of the approach proposed in this report. It combines fracture mechanics based residual life prediction with Monte Carlo simulation.

The method described will provide a tool for the evaluation and optimisation of inspection procedures and action timescales, a means of predicting what effects changes in traffic pattern will have on breakage rates and a means of assessing the benefits of different forms of track construction.

The method is necessarily limited by the boundaries of current scientific knowledge. In particular the rate of growth of rolling contact fatigue cracks is still an active area of research. Hence, whilst there is no conceptual problem in applying the method to head checks and squats, in practice, our fundamental understanding of the phenomenon may need to be to be advanced before the approach can be applied.

In the form presented here, the model also assumes that the initial defect is small. Thus the model will not deal with the major defects that occasionally occur in alumino-thermic welds as a function of process faults. Arguably this is more a process control problem than one that should be contained by routine inspection.

In the next phase of the project, the methodology described will be implemented as a software package. This will be available to Infrastructure Managers under a separate agreement.

2. Introduction

The term ‘minimum action’ is used in a railway context to define actions that the engineer with responsibility for the safety of the line must take in the event that a defective or broken rail is discovered[1].

Historically ‘Minimum Action’ rules have developed pragmatically. As a result, national practices reflect the historical experience of individual railway administrations. Unsurprisingly this has resulted in a diversity of practices across Europe.

This diversity implies, potentially, an implicit acceptance of different levels of risk, presently undefined. Perhaps more importantly, the empirical nature of the rules also means that there can be no certainty that when traffic characteristics change, for example speeds or axle loads are increased, what has proved satisfactory in the past will continue to be so.

One objective of SP4, within the Innotrack project, is to develop a methodology for the determination of 'Minimum Actions' on a scientific basis. Such a methodology is described in this report.

The process put forward is complex and requires data on a wide range of factors to be input. The benefit of this however is that the effect of that same wide range of factors on 'Minimum Action' timescales can be assessed.

The process is based on the modelling of the crack growth process under the cyclic loading produced by traffic rather than on statistical correlations. The advantage of going back to fundamentals in this way is that it is possible to look at radically different situations and solutions to those that currently exist.

In the next phase of the project, the methodology described will be implemented as a software package. This will be available to Infrastructure Managers under a separate agreement. Once developed and implemented it will

- enable convergent standards to be developed that represent a comparable level of risk

and

- provide a rational basis for rules under new operating conditions

There is also the potential, that in some circumstances a scientific approach will justify a relaxation of 'action' time scales, thus enabling action to be taken as part of a planned maintenance cycle, rather than as an emergency measure, at a considerable saving in cost.

[1] The term ‘minimum action’ is used to emphasise that the engineer may take additional action should he or she deem it necessary or desirable.

3. A Scientific Basis for 'Minimum Actions' for Defective Rails

3.1 The Principle

In principle, the issue is a simple application of fracture mechanics. However, as will become evident, the task is one of considerable complexity.

The fundamental assumption underlying Linear Elastic Fracture Mechanics (LEFM) is that the conditions at any point at the tip of a crack can be characterized by a ‘stress intensity factor’, K, derived from an elastic stress analysis. The assumption is valid provided that the plastic zone at the tip of the crack is small compared to the overall size of the crack (Irwin, 1957)[2,3].

Its consequence is that two cracks, in the same material, when subjected to the same stress intensity factor history, can be expected to behave in the same way. Thus, if, in the laboratory, a cracked test piece is cyclically loaded such that the stress intensity factor history replicates that experienced by a crack in a trafficked rail, the two cracks can be expected to grow at the same rate.

Two empirical developments of LEFM are of major importance. Paris et al. (1963), showed that the crack growth rate (da/dN), where ‘a’ is the crack size and ‘N’ is the number of applied load cycles could be related to the stress intensity factor range (ΔK) experienced by a comparatively simple relationship (Paris’ law):

$$(da/dN) = c.\ (\Delta K)^n$$

'c' and 'n' being material dependent.

The second observation was that there existed a lower bound value of K at which brittle fracture might occur, usually designated K_{1c} and referred as the ‘plane strain fracture toughness’ of the material. Thus if the cyclic maximum value of K (K_{max}) exceeds K_{1c}, there is a possibility of immediate breakage.

The stress intensity factor is a defined function of crack size and the applied forces or stresses, i.e.

$$K = F(\sigma, a)$$

A knowledge of any two (plus a knowledge of the form of F) enables the third to be estimated. Thus if σ and K_{1c} are known, the minimum crack size (a_f) at failure can be estimated[4].

Integration of Paris’ law enables the crack size after N cycles to be estimated:

$$\int da /(c.(\Delta K)^n = \int dN$$

[2] All references, including those in the appendices, are listed at the end of the report.
[3] For rail manufacture, steels require a combination of reasonably high strength and high wear resistance at an economically viable price. These requirements leads to the almost universal use of pearlitic steels for rail manufacture. Pearlitic steels are of relatively low toughness. The combination of a moderately high yield strength (restricting the size of the plastic zone) with a low toughness (limiting the maximum applied stress intensity factor) means that LEFM can be applied to the majority of problems involving cracked rails.
[4] This is the simplest case. If more than one loading system is involved, the form of 'F' will be different in each case and the value of K needs to be determined by adding the contributions from each source together.

If the current crack size in a rail is known (a_0), and the crack size at which the rail may fail is also known (a_f), these can be set as the integration limits and the number of load cycles to failure (N_f) predicted[5]. Thus from a knowledge of the current crack size, the loading conditions, the Paris' law parameters (c, n) and the fracture toughness of the rail steel, the residual life of a cracked rail can be estimated and thus the time frame determined for action to mitigate the breakage risk[6].

3.2 The Process

3.2.1 Introduction

Generally, because of the complexity of the 'K-calibration' or 'compliance function', F, the integration has to be performed numerically. Set out below is a process for the deterministic estimation of the residual life of a cracked rail. The elements of the process are discussed in more detail in subsequent sub-sections.

[5] Other, more complex, equations have been developed to describe the relationship between crack growth rate and the stress intensity factor history, but the principle outlined above remains valid.

[6] It is for this reason that the European Rail Standard sets out minimum values for K_{1c} and maxima for da/dN at specified values of ΔK

3.2.2 Procedure

Data Input

Descriptive material - to enable the output to be identified with the application

Live stress data It is assumed that the live stress history has been preprocessed to create a set of stress range/ mean stress/ number of occurrences triads. The data required are the number of triads, and the range, mean and occurrence values

Conversion factor The live stress history will represent a finite period such as 24hours track usage or 0.1MGT. A factor is required to convert 'life' in terms of the number of times the stress history has been applied to 'life' in terms of time or tonnage

Stresses due to static & quasi-static loading. This will include residual stresses, and in the case of rails, thermal stresses and 'rolling out stresses'

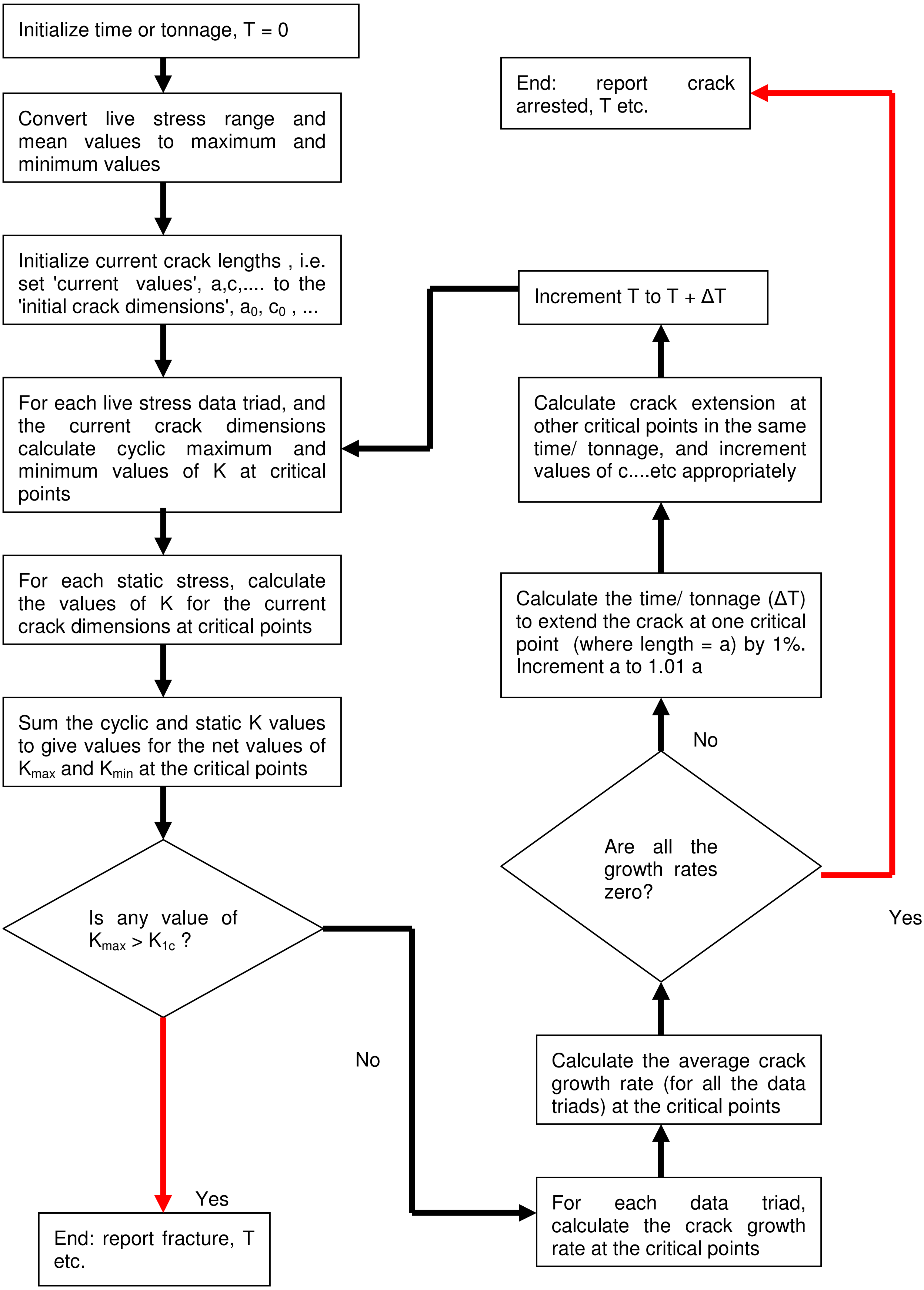
Geometry The inputs need to describe the crack geometry, the initial dimensions of the crack, and possibly any limiting values that are to be assumed and the critical dimensions of the body. Information to describe the stress distributions associated with the live and static stresses is also required.

Compliance functions For each loading case, static and dynamic, it is necessary to establish a value for the resultant stress intensity factor. Compliance functions can be created as an iterative loop using finite element or boundary integral methods as the crack develops but if solutions can be entered in a closed form or as a look up table, this is much to be preferred.

Materials data In general, the following are required.

The fatigue crack growth threshold, and its variation with stress ratio, the 'Paris Law' parameters (or, if another law is used, an equivalent set of values), and the fracture toughness, K_{1c}

Flow Chart


Initialize time or tonnage, T = 0
Convert live stress range and mean values to maximum and minimum values
Initialize current crack lengths , i.e. set 'current values', a,c,.... to the 'initial crack dimensions', a_0, c_0 , ...
For each live stress data triad, and the current crack dimensions calculate cyclic maximum and minimum values of K at critical points
For each static stress, calculate the values of K for the current crack dimensions at critical points
Sum the cyclic and static K values to give values for the net values of K_{max} and K_{min} at the critical points
Is any value of $K_{max} > K_{1c}$?
Yes
End: report fracture, T etc.
No
For each data triad, calculate the crack growth rate at the critical points
Calculate the average crack growth rate (for all the data triads) at the critical points
Are all the growth rates zero?
Yes
End: report crack arrested, T etc.
No
Calculate the time/ tonnage (ΔT) to extend the crack at one critical point (where length = a) by 1%. Increment a to 1.01 a
Calculate crack extension at other critical points in the same time/ tonnage, and increment values of c....etc appropriately
Increment T to T + ΔT

3.2.3 Stress Data

The following need to be taken into account.

Live Stresses

The live stresses due to traffic may include stresses associated with the overall bending of the rail, local bending stresses (as when, directly beneath the wheel, the head bends as an independent beam on an elastic foundation) and contact stresses.

The data may initially be in the form of a stress history for a specified period at some reference surface on the rail. Information also needs to be input that enables these surface stresses to be related to the stress field within the rail.

More generally, a stress history will need to be constructed from predictions of the wheel-rail forces, the rail's properties and the track support stiffness[7].

Track quality is conventionally measured as the standard deviation of the top or line over a specified distance, usually of the order of 200m. It is often supposed that a poor track quality equates to high track forces. However standard deviation is a poor indicator of dynamic track forces as it is independent of wavelength.

Long range imperfections have passenger comfort implications, but, as a generalisation, they do not result in a significant increase in the wheel/rail force. On the other hand short range irregularities, such as a dipped or stepped welds, can generate high forces. Speed becomes highly significant not only because dynamic forces are speed dependent but also because the point at which the wheel/rail force reaches a maximum is speed and support stiffness dependent.

Away from short range top faults, the static axle load can be taken as a reasonable starting point for the prediction of stresses. Where short range faults exist a more complex analysis is required. As a matter of observation, there is also, generally, a large amount of scatter in the stresses realised close to a discrete track irregularity.

Wheel irregularities have similar effects to track irregularities with the proviso that whereas the impact associated with a track irregularity will always occur at roughly the same place (for a given vehicle, speed etc.), that associated with a wheel irregularity will occur randomly. In general it has therefore been historic practice to ignore the contribution of wheel irregularities to the development of cracks in rails. However while they are likely to have a negligible contribution to the growth rate, wheel flats can truncate the life of the rail by inducing brittle fracture.[8]

The stress history may be pre-processed by a cycle counting algorithm to give pairs of stress range and mean stress values, each of which is associated with the number of times it occurs in a representative period, for example a day, a week or per megatonne of traffic. Alternatively a 'cycle by cycle' growth calculation may be undertaken but this does not appear to offer any advantage in this application.

Thermal Stresses

In CWR the rail is prevented from expanding and contracting as the temperature changes, resulting in the development of 'thermal stress'. To minimise the risk of buckling in hot weather, it is standard practice to install the rail so that the thermal stress is approximately zero when the rail temperature is in the mid 20's °C. At temperatures higher than this 'stress free temperature', SFT, the thermal stress is compressive, and at lower temperatures it is tensile.

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[7] In some situations, mostly obviously when considering the development of head cracks in rails on curves, lateral loading may also need to be considered.

[8] Wheel flats can cause severe damage to concrete sleepers and a wheel impact control strategy was first introduced in the UK to counteract this. Using the type of model proposed in this report, it is possible to look critically at the effect of such strategies on rail breakage risk.

The SFT, can however be significantly modified by:

- Wholesale slewing of the track towards the centre of a curve in cold weather
- Rail creep due to the wheel guidance forces on curves
- Rail creep due to traction and braking forces, particularly on banks
- Ground movement, for example due to mining subsidence

Under certain circumstances, thermal stresses can also reach significant magnitudes in track with traditional bolted joints.

Residual Stresses

Residual stresses are of critical importance not only from the point of view of calculating residual life, but also because of the major influence they have on crack initiation. As an example, the abrupt change in section associated with an alumino-thermic weld must create a severe stress concentration at the weld toe. Despite this, metallurgically sound welds have good fatigue strength when subjected to repeated bending. This is attributable entirely to the compressive longitudinal residual stresses generated in the head and foot of the rail during the welding process.

Rolling Out Stresses

Wheel-rail contact has the effect of ‘rolling out’- extending - the top surface of the rail. Except at the extreme rail ends, this extension is resisted by the body of the rail. The net effect is to modify the residual stress distribution and to increase the free rail length. In CWR, this overall expansion is prevented which results in the development of a uniform compressive stress in the rail. However when the rail is removed from track the rail lengthens and the residual stress field is partially relaxed. In the case of roller straightened rails, the effect of this relaxation can be seen as the rail bends with the head extending and the foot shortening.

These elements of the residual stress field need to be added to those measured conventionally to give a full picture of the stress field experienced when the rail is in track.

3.2.4 Geometry Data

The initial size of the crack needs to be defined along with the key dimensions of the rail and the location of the crack within it.

3.2.5 Materials Data

The materials data required are normally the plane strain fracture toughness, the fatigue crack growth threshold and its variation with the stress ratio, R, and crack growth rate data in what is usually referred to as the ‘Paris region’ because Paris’ law applies.

In some circumstances, for example if rules were being developed for a high toughness material like austenitic manganese steel, additional data on yield properties might be required.

3.2.6 The Compliance Function

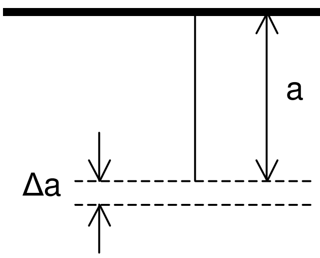
This defines the relationship between the applied loads or stresses, the crack size, and the value of K, the stress intensity factor. For a number of crack types, for example cracks at bolt holes, semi-elliptical cracks growing perpendicular to a flat surface (such as a foot crack in plain rail) published solutions are available which can be accessed either via closed form equations or look-up tables.

In general few solutions are available for other than planar cracks. In principle new solutions can be generated by, for example, finite element (FE) and boundary integral (BI) methods. However for surface breaking rolling contact fatigue cracks, where the 'plastic wake' of deformed material on the walls of the crack could affect the volume of fluid trapped as a wheel passes over and where loading rate and viscosity could affect the pressures generated by partially entrapped fluid, one is into 'current research' rather than proven solutions.

3.2.7 Relating Crack Growth to Traffic

There are slight differences here depending on whether the live stress data have been pre-processed to produce a matrix of stress range, stress mean and number of occurrences per unit time or whether a cycle by cycle (effectively an axle by axle calculation) is performed. Conceptually the cycle by cycle approach is the more straightforward, it is potentially more flexible but it is also more time consuming.

For simplicity, consider a two dimensional situation.



The crack is initially of length 'a'. Under the action of the first axle, it is estimated to grow by an amount Δa_1, calculated using a knowledge of the range of K associated with the load cycle and the applicable crack growth law. One now calculates the values of K and the growth rate for the second axle, noting that the crack length is now no longer 'a', but '$a + \Delta a_1$'. The increment of growth associated with this axle can then be calculated, Δa_2. This process is repeated until the crack reaches such as size that the fracture toughness is exceeded and fracture is predicted, a record of the number of axle passes or days or megatonnes of traffic simulated being maintained throughout.

If the stress history has been pre-processed to create a matrix of pairs of stress range & mean values and of their number of occurrences within a specified period (such as a day), the process is slightly altered. One considers a small increment (Δa) of crack growth (one where the geometry change is sufficiently small for the change in the compliance function to be negligible). One estimates the growth expected under a day of traffic based on the current crack length then determines how many 'days' of traffic are required to extend the crack by this amount. One then increments the crack length by this small amount and repeats the process, maintaining, as one goes a running total of the number of days. As previously, the calculation is terminated (and the life in 'days' reported) when either the fracture toughness is exceeded, or a prescribed crack length reached.

An example is worked through in more detail in Appendix A. As the purpose of the annexe is to demonstrate principles, the case studied has been deliberately chosen for its relative simplicity rather than its technical importance.

3.3 Probabilistic Considerations

Suppose one knew exactly the initial size and shape of a crack that had been detected in a rail, one knew exactly the stress history to which the rail would be subjected, residual stress distribution in the rail, the material properties of the rail etc. then one could predict with a very high confidence the residual life of that specific rail.

Consider however the reality. In every area where data are required there is uncertainty. For example, just one factor that will influence the predicted life is the crack growth rate law assumed. Whilst the evidence is that the rates of growth in rail steels do not vary significantly as a function of the source of supply (thus for example all grade 260 steels behave in much the same way) it has long been recognized that there is an inherent scatter in the results of crack growth tests. A factor of 2 between replicate tests is not unusual (BSI, 1988). This means that the residual life of one cracked rail could be half or twice that of a nominally identical rail.

In determining the timescales for 'minimum actions' one could take an 'upper bound' approach, that is take the most pessimistic view of the likely crack growth rate. However this, of itself, creates problems. An 'upper bound' crack growth relationship does not really exist. There will always be a small but finite probability that whatever growth rate is assumed will be exceeded. Implicitly one is accepting a small but finite risk that the prediction will be unsafe. Further, if one extreme assumption is combined with others, one ends up with an extremely conservative life prediction. The minimum action timescale might thus be predicated on the basis of a combination of circumstances that might happen once in 100 years. The likelihood is that these would be impractically onerous for the railways to implement.

A scientific approach requires that minimum actions are based on a proper evaluation of risk. 'In Monte Carlo simulation, a logical model of the system being analysed is repeatedly evaluated, each run using different values of the distributed parameters. The selection of the parameter values is made randomly, but with probabilities governed by the relevant distribution functions' (O'Connor, 1991).

In this case, the 'logical model' is a deterministic residual life prediction model, as outlined in section 3. The output is a statistical distribution of residual life and the risk of rail breakage as a function of time or tonnage and thus of the risk associated with specific inspection regimes and minimum action timescales.

As a method of dealing with uncertainties, the Monte Carlo method has the great advantage of being conceptually straightforward. It is also generally the most efficient way forward in a situation as complex as that under consideration, where so many variables have to be taken into account. A simple worked example is given in Annexe D.

3.4 Data Input for a Probabilistic Model

3.4.1 Stress Data

Stress data will not be available for the vast majority of cases, and clearly never will be when the effect of future traffic patterns is under evaluation. One's starting point is therefore, of necessity, a knowledge of static axle loads. From this one can create a representative history for say 24 hours of operation.

Track categories are usually established on the basis of annual tonnage or a weighted tonnage figure. These may be made up in a variety of ways. For example 15 MGTPA (million gross tonnes per annum) could represent 40 heavy freights per day or a high density suburban service. Thus an analysis based on annual tonnage would need to take account of the variety of traffic patterns represented on the system in this category and the fraction of the system where a specific traffic pattern applies.

Loads can be converted to stresses via a beam on an elastic foundation model. Clearly rail section will be a critical factor in this conversion, as will support stiffness.

How one deals with these aspects will depend on the application. However, in Britain, for example, inspection intervals are simply tied to traffic levels and neither they, nor the 'minimum actions' vary with the track construction. One has the option of looking at how track construction might affect timescales but one can also accept the status quo, in which case the mix of construction types considered should match that in reality. Thus if, in a particular track category, 70% of lines utilize bull-head rails with timber sleepers, and 30% 56E1 rail on concrete, these percentages should be reflected in the sampling process used to generate a track stiffness value and thus to convert a load history to a stress history.

Some types of defect are directly associated with track top irregularities, cracks at fish bolt holes being a prime example. In this case, the local track geometry needs to be taken into consideration. To do this on a system wide basis will require appropriate measurements to be made by a track recording car, so that a population of 'equivalent dip angles' can be generated, and then sampled for specific runs of the deterministic model. Unsprung mass information will also be required.

As a matter of observation, there is a scatter in the stresses measured over and above that expected from simple models. This could come from uneven loading of a vehicle or just the vertical motion of the wheelset as a function of its interaction with the track over which it has just passed. At the moment there seems no better way to deal with this than to incorporate an empirical 'scatter factor' on load.

3.4.2 Geometry Data

The objective is to determine the residual life of a cracked rail from the moment a defect is detected. However in most instances the inspection systems in current use merely flag the existence of a crack; there is little information about the crack size at detection.

This problem can be approached in two ways. For existing crack types, samples can be collected to determine the crack size distribution at detection. This would enable the probability of failure associated with changes in 'action' timescales to be evaluated.

This method however is limited in its application as the crack size distribution itself will be affected by the inspection method and periodicity and possibly by changes in traffic.

A more flexible approach (Beagles, 1996) requires not a knowledge of the crack size at detection but of the probability of detection of a defect as a function of its size[9].

With the exception of some cracks growing from pre-existing defects, all cracks start 'small'. In most cases, in their early stages of development, they are undetectable using current techniques.

At some arbitrary time, we can therefore say that a crack will have dimensions 'a_0, c_0' and will be of such a size that it will not be detected. If we take those as our initial crack dimensions, we can predict the growth of that defect from that point on[10].

However to determine the residual life from the moment of crack detection, we still need to know the crack dimensions at the time the crack is detected.

Suppose the inspection period is 'P'. The first inspection could fall randomly at any time between time '0' (the start of our crack growth prediction) and 'P'. In keeping with the principle of Monte Carlo simulation we therefore select randomly a time 't' between 0 and P for the timing of the initial inspection.

At this point, our prediction indicates that the crack has reached a size of a_1, c_1.

At this point also, the defect may or may not be detected, this being determined from the probability of detection curve[11]. For example, suppose the curve indicates that the probability of detection, for the given crack size, is 33%. One can construct an array with say 100 elements. Into 33% of these elements one stores 'defect detected' and in the remainder 'defect not detected'. One samples the array at random to determine whether in this simulation the defect will be 'detected' or not.

If, in the simulation, the defect is detected, then the residual life is estimated from this point. If it is not, then the crack is allowed to grow for a further period 'P' and the process repeated. This is continued until either the defect is detected or breakage is predicted.

Rail section loss by corrosion and wear is generally slow, so one is justified in using constant 'rail dimensions' during any particular deterministic calculation. However, particularly for head defects, one may need to take account of varying wear levels.

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[9] The expected distribution of defect size at detection is, in fact, an output that can be obtained using this approach.

[10] A reasonable initial assumption is often that the crack is circular, semi-circular or quarter circular (i.e. that $a_0 = c_0$). Whilst this may not be strictly accurate, the growth simulation process should correct any initial errors concerning the crack shape as the simulated defect develops.

[11] The curve representing the probability of detection as a function of crack size will usually be based on 'expert opinion'. The approach allows the effect of changes in the assumed curve to be evaluated, to see whether they have a significant effect. Indeed the effect of a complete change in inspection method, which results in a completely different probability of detection curve, can be assessed.

3.4.3 Materials data

The 'Paris region' Crack growth law

Typically, the crack growth rate test data will have the form shown in Fig. 2, i.e. the results from a series of tests can be represented by a series of nearly parallel lines. One could, for a given value of ΔK establish a mean and standard deviation for the growth rate, then in the context of the calculation, sample this distribution at random in the life prediction. This however would be misleading as one element of the life calculation could be based on the blue curve and another on the green. In reality the rail specimen will follow the blue curve, or the green or the black or the red etc. It will not jump between them. The option suggested is to establish a library of crack growth curves, and select one curve, at random, from the library for each life calculation.

Crack growth thresholds

Consider again Figure 2. The threshold associated with the blue line will differ from that associated with the green line. If the 'blue' threshold were associated with the 'green' Paris line an erroneous picture would be created of the near threshold crack growth characteristics of the sample represented by the green line. It is thus more realistic to maintain the 'library' approach, linking specific thresholds with the Paris line for the same sample.

The strong dependence of the threshold value on stress ratio should be noted.

Fracture toughness

Test values obtained at a given loading rate and temperature appear to be normally distributed. The toughness of pearlitic rails shows a slight temperature dependence and there can also be strain rate dependence, but at the very high loading rates of relevance to rail service the steel is firmly on the lower shelf, and neither effect is worth worrying about.

Early work on ingot cast rail steels (Cannon et al. 1972) showed that web material was less tough than head material. Their results also showed that the toughness was little affected by composition (within the range then permitted by BS 11), rail source, position in the ingot and manufacturing process. It is not known whether a study of comparable thoroughness has ever been undertaken on continuously cast steel, but a similar conclusion appears likely.

Rail grade

There is undoubtedly a small influence of rail grade on toughness, but, despite the plethora of data (see, for example, Toth, 2000), it seems quite difficult to determine whether the fatigue crack growth characteristics are significantly influenced by grade. Where rail grade influences are a concern, then the material properties can be selected in a way that reflects the relative amounts of the each grade installed in track. One could also assess whether a wholesale change of grade would have a significant effect.

It should be noted that rail grade and the rail production route can both affect residual stresses, with potentially significant effects on residual life.

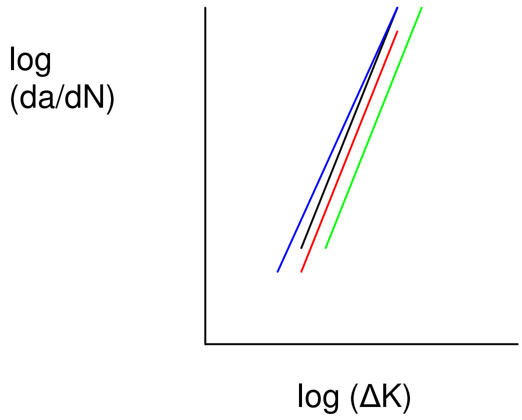

Fig.2 Simplified schematic diagram of crack growth rate - ΔK relationship

3.5 Other Factors

3.5.1 Derailment Consequences

The foregoing has addressed the issue of the probability of breakage as the driver for Minimum Action timescales. However the consequences of rail breakage may justify a difference in response in different situations. Most obviously the consequences of a high speed derailment are likely to be radically more severe than a derailment at low speed. It is important however not to complicate the situation to the point where mistakes are likely to be made 'on the ground'.

3.5.2 Inspection Limitations

If a rail is deemed 'uninspectable' by reason of its surface condition, then there can be no assurance of its continued serviceability. Safety can only be assured by acting as though it presents an immediate risk to traffic. The much publicized 'Hatfield derailment' in Britain in 2000 can be attributed to a failure to recognise this principle.

There are also defects, for example foot cracks in plain rail, which are difficult to detect until they are of such a size that they already represent a breakage risk. Consequently, the breakage risk cannot be controlled adequately through inspection using current technologies however frequently inspections are carried out. Another approach has to be adopted. In this particular instance a 'safe life' approach has been suggested (Allen, 2004).

The non-destructive inspection of alumino-thermic welds is possible but very time consuming using current techniques. In addition these techniques do not lend themselves to vehicle based testing. In this case it is also necessary, for the moment, to look for other methods of ensuring reliability. In Britain the historic approach was to place a great emphasis on welder training and on the periodic re-testing of welders. This provided assurance of the continued competence of the welders and through this, an assurance of the soundness of the welds they produced.

3.5.3 Key Defects

If there were no inspection, then the defect types of most concern would be those which, by virtue of their frequency of occurrence and the likely consequences of an associated rail break, would be regarded as creating the highest level of risk to traffic.

It is suggested that these high risk defect types should be the first concern in this project. Collation of rail failure statistics throughout Europe will enable the most frequently occurring defect types to be identified. Collation of accident reports could also enable a picture to be built up of the consequences of particular types of breakage.

3.5.4 Uncertainties

There will be instances where the quality of information available is poor. For example, whilst a number of factors have been identified as affecting the stress-free temperature[12] (SFT), the proportion of sites, and the extent to which they may have been affected by these factors is unknown. It is legitimate to ask the question 'Does this matter?', a question which can be addressed by investigating the sensitivity of the life distribution predicted using the Monte Carlo method to various assumptions about SFT. If the life distribution proves to be insensitive to the assumptions made (within reasonable bounds), then one is justified in concluding that though the input information is of poor quality, it is adequate for the purpose. However, if a sensitivity is demonstrated, then one must recognise this and seek to improve the quality of data available.

3.5.5 Generic Vehicles

In the general context of track damage modelling, Innotrack sub-project SP1 'Duty' has set out to establish whether, across Europe, there is sufficient similarity between vehicles to enable a limited number of 'generic vehicle types' to be defined which adequately represent the European fleet, so simplifying the modelling of track geometry deterioration.

Once created, the model proposed in this report will enable the applicability of this concept to rail defect development to be evaluated.

3.5.6 ‘Acceptable Risk

The objective of the proposed work may be seen as the development of a scientific basis for 'Minimum Actions' so that for a given, acceptable, level of risk to the travelling public, railway staff and goods in transit, an economically optimised inspection, maintenance and 'action' regime may be established. The travelling public however perceives the railway to be 'safe', hence, to them, and to politicians, no level of risk is acceptable. As a consequence, there are constraints on what can be proposed.

If there is no such thing as an 'acceptable level of risk', then it must be demonstrated that what is proposed is at least as safe as current practice and preferably offers a reduction in risk as well as economic benefit. It follows that one element of the work must be an assessment of current risks. Additionally, the model used needs to be rigorous to the point where it could be defended in a court of law. A 'state of the art' model is not adequate if the 'art' is manifestly inadequate.

[12] When rails are installed in continuously welded track, they are generally pre-tensioned. The effect of raising the temperature of the rail is then to relax this pre-tension, rather than to put the rail into compression. Eventually however, the pre-tension will be overcome and the rail will start to go into longitudinal compression. The temperature at which this happens is referred to as the 'stress free temperature', abbreviated to SFT. A high SFT reduces the track buckling risk in hot weather but can encourage defect growth and rail breakage at low temperatures.

3.6 Implementation

3.6.1 Key Defects

These need to be defined and agreed, as described above.

3.6.2 Inspection Capability

For the key defect types, an assessment should be made of the scope for control using current inspection techniques. There is, for example, little to be gained by building a crack growth model for casting defects in alumino-thermic welds until such time as an adequate inspection regime exists.

3.6.3 Assessment of Available Data

If the risk can be controlled through inspection, the available information needs to be reviewed critically and a view taken on whether enough is known for a credible model to be developed. Inter alia, this will require consideration of crack growth laws applicable to the loading situation, residual stress distributions, crack detection probability and sizing capability.

It has to be accepted that, whilst there is no doubt that a framework model can be developed, some of the knowledge gaps may be too big to enable defensible results to be obtained. This process of data assessment will define these gaps but there may not be the resources within the Innotrack programme to obtain all the necessary information.

It would seem very likely that one key defect type will be the surface initiated transverse head crack. It would therefore be prudent to instigate a review of the available information on crack growth under sequential mixed mode loading at an early stage.

3.6.4 Development of a Framework Model

The first stage will be the development of a comprehensive flow chart. It is important that the program is structured in such a way that both the program itself, and the input data, can be modified relatively easily

3.6.5 Demonstration of Model Capability

Ideally this will focus on a 'key defect'. If this is impractical because of current inspection limitations or gaps in knowledge, then another case, to be agreed, will be used to demonstrate the model's capability

3.6.6 Evaluation of Current Risks

What is proposed for the future must be demonstrably as safe as current practice. In order that current risks can be assessed, a compilation of European 'Minimum Action' rules is required.

4. Conclusions

An approach has been described, based in accepted scientific principles, for the evaluation of 'Minimum Actions' in a railway context.

The method requires a considerable amount of data as input, and in some circumstances this will limit its application at the present time. It is however a very flexible method, providing a tool which has the potential to evaluate the influence of a large range of variables on the probability of rail breakage including changes in inspection method, changes in the periodicity of inspection, changes in traffic pattern, changes in track construction and changes in rail metallurgy.

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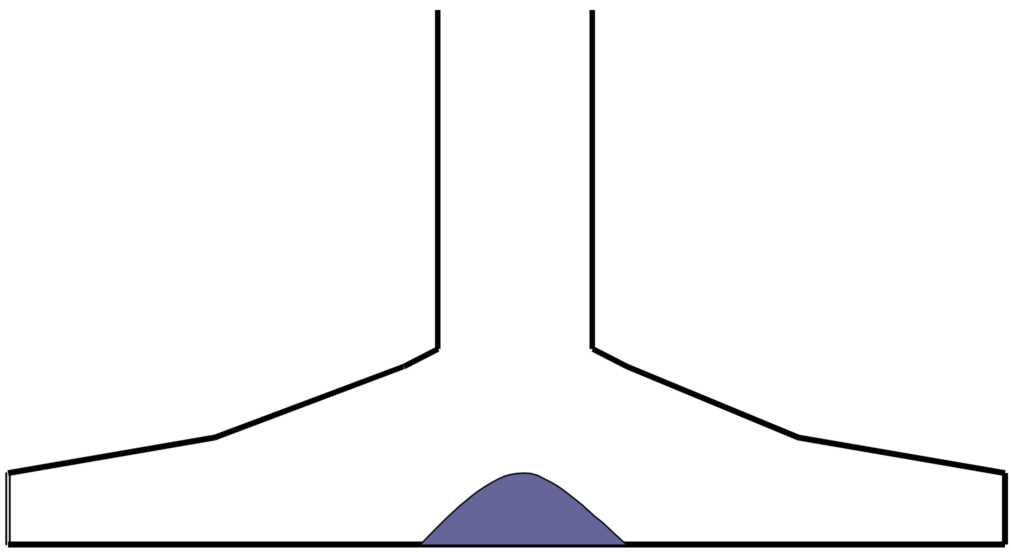
[13] This standard has now been superseded by BS ISO 12108: 2002, 'Metallic materials. Fatigue Testing. Fatigue crack growth method'.

6. Annexes

Annexe A Deterministic prediction of the residual life of a cracked rail

A.1. Introduction

This annexe sets out the process required to undertake a deterministic estimation of the residual life of a cracked body, specifically a rail containing a crack on the underside of the foot, beneath the web. This example has been chosen for its relative simplicity, from a fracture mechanics point of view, as the crack is simply subject to 'mode 1' displacements.[14] There is particular reference to British conditions, with which the author is most familiar, but the method can easily be applied elsewhere.



A.2. Stress Data

A.2.1 Traffic Loading

The rail may be treated as a beam on an elastic foundation (see Annexe B). Given a knowledge of the support stiffness (see Annexe C) and the rail height and moment of inertia, wheel rail loads may be converted into a stress history for the underside of the foot. With bogie vehicles it is generally necessary to take account of the interaction between the stress fields created by closely spaced wheelsets.

The stress history may, for convenience, be 'cycle counted' to produce a matrix of stress range, mean stress and number of occurrences values for a day's traffic. (Here a 'day' is used as the unit of time for residual life prediction: other options are 'megatonnes' or 'weeks').

To convert the surface stresses into a stress distribution, one also needs to know the rail height. This will therefore be a necessary input when geometry data are considered.

[14] Crack faces can more relative to one another in three 'modes'. Mode I involves the faces simply moving apart. Mode II involves them sliding over one another in a direction perpendicular to the crack front, and mode III involves them sliding over one another parallel to the crack front.

A.2.2 Thermal Loads

The stress distribution due to thermal loads will be uniform tension or compression throughout the rail section. The stresses due to thermal loads will vary at a much lower frequency than the loads due to traffic. Thus they will not directly cause fatigue damage, but will modify the damage caused by traffic loads.

In a deterministic calculation one can deal with thermal stresses in two ways. First, one can enter a single 'representative value'. Alternatively one could break the day's traffic into segments and associate a particular thermal stress with part of the day. Clearly further extension of this principle is possible, at the expense of increasing complexity.

Conventionally thermal stresses in CWR are calculated from a knowledge of the stress free temperature, nominally 22 - 27ºC in Great Britain. Thus

$$\sigma_{th} = E.\alpha.\ (S - T)$$

where

σ_{th}	=	Thermal stress
E	=	Young's modulus for rail steel, usually quoted as 207 - 213GPa
α	=	Thermal coefficient of linear expansion of rail steel, about 12×10^{-6} per ºC at ambient temperature
S	=	Stress free temperature (ºC)
T	=	Rail temperature (ºC)

Note that on cloudy days the rail temperature and the air temperature are usually comparable: on sunny days the rail temperature (in Britain) is often about twice the air temperature (both measured in ºC).

A.2.3 Residual Stresses

There are some data about residual stresses in the foot of roller straightened rails, but they are not extensive. The expectation is that they will be high and tensile at the centre of the foot (also the most common location for cracking), reducing to zero as the web - foot junction is approached. They will also fall off to a low value towards the edges of the foot.

These are static stresses, pure and simple. Thus, like the thermal stresses, they will not directly cause fatigue damage but will influence the damage due to traffic loads.

In this instance geometry data will need to be input that enable the stress distribution to be described, for example the distance from the underside of the foot to the point near the head web junction where the stress level approaches zero, and the foot width. A weight function type approach could then be used to calculate values of the stress intensity factor. Alternatively, in this application it is probably legitimate (with roller straightened rails) to ignore the lateral variation of residual stress and model the residual stress field as a simple bending field with the effective neutral axis at about the foot web junction.

A.2.4 Rolling out stresses

These can be modelled as a combination of a combination of uniform tension and bending stresses in the rail.

A.2.5 Geometry Data

From the discussion of the stress data inputs required it will be evident that the rail height, possibly the rail foot width and the distance to the web-foot junction will be required (in order to characterize the stress fields associated with the various loadings) in addition to the initial crack dimensions. In some instances one may also wish to input a maximum crack size. For example austenitic manganese rails have such a high toughness that brittle fracture is not normally experienced. When the crack reaches a certain size, the rail will 'fail' however by plastic collapse i.e. excessive deformation. A collapse algorithm could be written into the program, but alternatively it may be simpler to input a maximum crack size.

A.2.6 Materials Data

It is assumed that the form of the fatigue crack growth law is as follows

For $\Delta K \geq 1.5\ \Delta K_{th}$

$$da/dN = a.\ \Delta K^{b} \qquad (A.1)$$

It has been normal to assume that compressive stresses do not contribute to ΔK, hence ΔK is calculated from the tensile portion of the stress range only. This is consistent with limited British Rail Research (BRR) data on pearlitic rail steels under constant amplitude conditions. However later work using a broad band stress input indicated that growth rates were achieved in excess of those expected on this basis, the effect being that which would be expected if the crack remained open to a stress level -1/3 times that of the numerical value of the tensile stress peak. No closure measurements were made to confirm whether this was what was happening in reality and the nominal stress ranges used were determined by Rainflow counting (Smallwood, 1991).

For $\Delta K_{th} \leq \Delta K < 1.5.\Delta K_{th}$

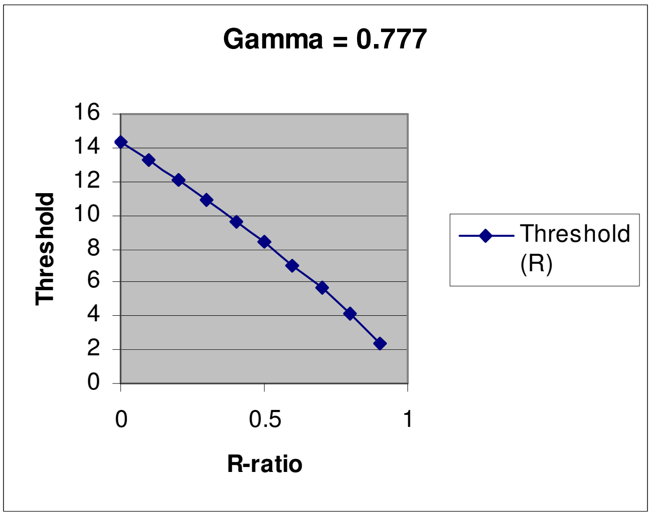
$$da/dN = c.(\Delta K - \Delta K_{th})^{d} \qquad (A.2)$$

The values of c and d are calculated so that the values of da/dN and the slope of the da/dN (ΔK) curve at $\Delta K = 1.5.(\Delta K_{th})$ given by eqn.(A.2) match those predicted by eqn.(A.1). A.1 is the standard form of Paris' law. A.2 was derived at British Rail Research by looking for the optimum data fit for near threshold crack growth in rail steel test pieces.

ΔK_{th} is assumed to depend on the value of R according to the following equations.

For positive values of R,

$$\Delta K_{th}(R) = \Delta K_{th}(0).[1 - R]^{\gamma} \qquad (A.3)$$

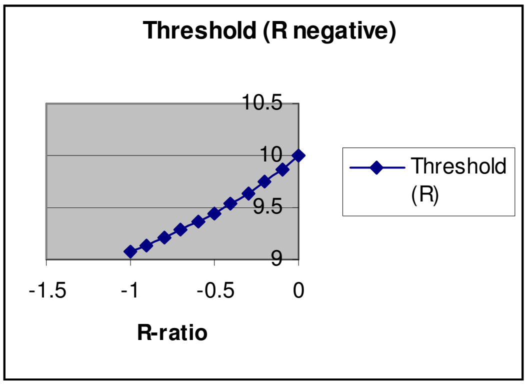


This is again based on unpublished work at British Rail Research on both rail steels and a structural steel. For 'grade 'A' (260) rail head material a value of γ = 0.777 fits the data well. However the highest value of 'R' at which tests were conducted was 0.73, so the form of the curve at still higher values of 'R' is conjectural. A linear curve, with the threshold taking a value of 2 MNm$^{-1.5}$ at R = 1, provides almost as good a fit to the data

For negative values of R, the following relationship is assumed.

$$\Delta K_{th}(R) = \Delta K_{th}(0)/[1 - R]^{0.14} \qquad (A.4)$$

which gives a 10% reduction in ΔK_{th} at R = -1, compared to the value at R = 0



This is consistent with British Rail Research data for R = 0 and R = -1, again for 'grade A' rail head material, but no tests were undertaken at intermediate values of R, or at values less than -1.

The significance of ΔK_{th} in a situation where some, but not all load cycles produce stress intensity factor ranges that exceed ΔK_{th}, has not been fully explored. Tests at BRR however showed that the effect of relatively frequent cycles exceeding the threshold level in a sequence including cycles below it had the effect of 'activating' the sub-threshold cycles, the overall crack growth rate being that which would have been expected if the Paris line simply continued down to lower values of ΔK. With infrequent cycles above the threshold, the growth was however lower than that which would have been expected from the 'above threshold' cycles alone. On this evidence a conservative but frequently accurate approach is to assume a zero threshold. However this is an area where additional research is desirable.

Where cracks develop under mixed mode loading, an 'equivalent crack growth law' could be used which takes account of the nature of the loading. Again this is a poorly researched area. So far as the author is aware, no one has looked at the effect of the combination of mixed mode and variable amplitude loading on crack growth. It's a daunting task because of the large number of variables involved.

Brittle fracture is assumed to occur if K_{max} exceeds K_{Ic}. The fracture toughness of conventional pearlitic rail steels is such that the risk of plastic collapse can be ignored. In the author's experience this also tends to be the case with bainitic rail steels. Austenitic Manganese Steel (AMS) however will normally fail by plastic collapse.

The following materials data are therefore required

- The Paris law constant and exponent, a and b, (Eqn. A.1). Note that it may be necessary to consider whether the data have been generated at an appropriate frequency and in an appropriate environment[15].
- Fatigue crack growth threshold when R = 0 (Eqns.A.3 and A.4)
- Gamma (Equation A.3)
- K_{Ic}

In some instances it needs to be recognized that some stress spectra are non-damaging. For example, in the case of a crack at a fish bolt hole, cracking may initiate not develop if stress levels are too low. In these case additional, conventional fatigue limit data will be required, particularly if a zero threshold is assumed, so that a finite life is not erroneously predicted.

A.2.7 Calculating K

A variety of options are available, but it should be born in mind that repeated calculation is required hence, in the interest of speed, data input in the form of look up tables or closed form solutions may be preferable and this is the approach assumed here. In this instance, the crack is semi-elliptical, a situation for which compliance functions for constant and linearly varying stresses in plates of finite thickness (T) are available.

The table below (Table1) indicates how these might be used.

For each peak and trough, the contribution of K from each of these forms of loading, whether positive or negative, needs to be calculated and summed. This is necessary whether a cycle by cycle damage sum is undertaken or the live (traffic related) stress data preprocessed by cycle counting.

Type of loading	**Stress distribution to be assumed**	**Value of 'T'[16]**	**Comment**
Wheel loads	Bending	T = rail height	In roller straightened pearlitic steels, the size of the crack is generally small compared to the overall size of the body. Hence the presence of a free surface at the top of the foot will have little effect on K.
Thermal stress	Uniform tension or compression	Not applicable	

[15] The values of 'c' and 'd' (Eqn. A.2) can be derived from the values of 'a' and 'b' by equating the values of da/ dN and the slopes of the curves defined by equations A1 and A2 when $\Delta K = 1.5\ \Delta K_{th}$

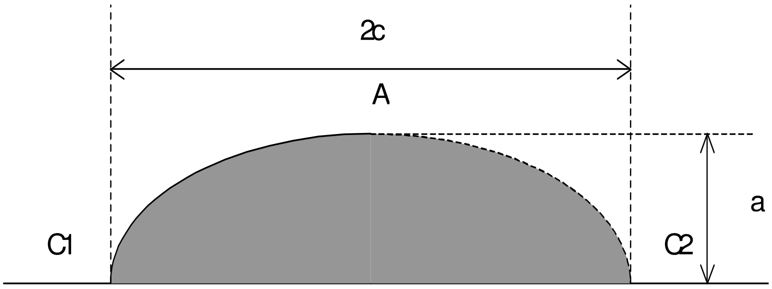
[16] 'T' is chosen to give the correct stress gradient.

Type of loading	**Stress distribution to be assumed**	**Value of 'T'[16]**	**Comment**
Residual stresses	Bending[17]	T = twice the distance to the web - foot junction	This gives a stress varying linearly from the outer fibre of the foot to zero at the web-foot junction. The lateral variation of the residual stress is ignored: this is reasonable in normal roller-straightened pearlitic steels as these crack do not extend a great distance laterally before fracture occurs
Rolling out stresses - rail extension component	Uniform stress	Not applicable	
Rolling out stresses - rail bending component	Bending	T = rail height	

Table 1 Synthesis of stress intensity factors for a rail foot crack

[17] It can be shown that the stress intensity factor due to a residual stress field is only dependent on the stress field that which would have existed had the crack not been present (Tada et al., 2000).

A.3. Growth Rate Calculations



In the case of a cycle by cycle damage approach, the maximum and minimum values of K at point A are first calculated. If the maximum value of K (taking account of all the applied stresses) exceeds K_{1c}, then fracture is predicted and computation ended. Otherwise the range of K, and the ratio of the minimum to maximum value will be required in order to estimate the growth using the equations given above.

In an elastic body, a negative value of K cannot exist because it implies a closed crack, and a closed crack creates no disturbance of the stress field. Conventionally therefore, if the minimum value of K, as calculated above, is negative, it is taken to be zero for the purposes of calculating the range of K[18]. Thus for

$$\Delta K = K_{max} - K_{min} \qquad \text{for } K_{min} > 0$$

$$\Delta K = K_{max} - 0 \qquad \text{for } K_{min} < 0$$

From the values of ΔK and R at point 'A', the increment of growth expected there is calculated, then in a similar way, the increments expected at points C1 and C2. The crack dimensions are then updated and the calculation repeated.

If a cycle counting procedure has been applied, it is usually convenient to calculate an average crack growth rate for point A and then determine how many time units are required to grow the crack a small distance. An increment of 2% of the existing crack length has been found to be adequate. An average growth rate for points C1, C2 is then calculated and the amount of growth expected in the same time frame determined. The crack geometry is then updated as previously and the calculation repeated.

Throughout the process the passage of 'time' is logged.

[18] It should be noted however that, when interpreted in terms of nominal values of ΔK, as calculated here, the da/dN curve for rail steels shows some dependence on the ratio of K_{min}/K_{max}

Annexe B The Rail as a Beam on an Elastic Foundation

Despite appearances, the rail generally behaves as a beam on an elastic foundation. Bending moments can thus be estimated from the equation (Roark, 1965):

$$M = [- P/ 4\beta]. [\exp(-\beta x)] . [\sin(\beta x) - \cos(\beta x)]$$

where P = the wheel rail force

- x = distance from the point of application of the force
- $\beta = [k/(4EI)]^{0.25}$
- E = Young's modulus for the rail
- I = Moment of inertia of the rail
- k = support stiffness per unit length

Note that when wheelsets are closely spaced the stress distributions created by individual wheelsets will overlap. The sag under the individual wheels is usually only slightly affected but the rail hog (and the associated stresses) between wheels is enhanced.

Annexe C Rail Support Stiffness

Stiffness is conventionally defined as force divided by resultant displacement. In the context of track, a figure often quoted is the 'stiffness per sleeper end'[19]. Figures that have been regarded as typical in Britain are as follows:

Concrete sleepers	42 MN/m
Timber sleepers	27.8 MN/m

The figure depends on the type of sleeper because the sleeper itself is a beam on an elastic foundation. Hence the deflection at the rail seat is a function not just of the ballast and rail pad stiffness but also of the stiffness of the sleeper as a beam. k, the support stiffness per unit length, is simply the stiffness per sleeper end divided by the sleeper spacing. Hence a stiffness per sleeper end of 42 MN/m, combined with a sleeper spacing of 0.7m gives a value of k = 60 MNm^{-2}.

Work at British Rail Research however demonstrated that the support stiffness itself is load dependent and also shows considerable scatter (Watson, 1991). The following relationships have been quoted (Hamilton and Watson, 1995).

Timber sleepers

k (mean value, MNm^{-2}) = 0.637 P (axle load in tonnes)

standard deviation equivalent to a factor of 1.99

Steel sleepers

k (mean value, MNm^{-2}) = 1.037 P (axle load in tonnes)

standard deviation equivalent to a factor of 1.95

Concrete sleepers

k (mean value, MNm^{-2}) = 3.94 P (axle load in tonnes)

standard deviation equivalent to a factor of 1.5

It may be noted that at zero load, the effective stiffness is also zero

[19] The 'stiffness per sleeper end' is the stiffness that would be estimated by loading one rail seat and dividing the measured deflection.

Annexe D Monte Carlo Simulation

D.1. Introduction

The purpose of this annexe is to exemplify the process of Monte Carlo simulation. The question addressed is 'what is the likelihood of breakage occurring in rails with a mean toughness of 35 MNm-1.5, with a standard deviation of 3 MNm-1.5, when the combination of peak loads and crack sizes present creates a mean stress intensity factor of 30 MNm-1.5, with a standard deviation of 5. Both the fracture toughness and the applied stress intensity factor (s.i.f.) values are assumed to be normally distributed.

D.2. Worst Case Approach

Typically the mean ± 2 or 3 standard deviations value is used as the 'worst case'. In this case for the toughness this means 29 or 26 $MNm^{-1.5}$, and for the applied s.i.f., 40 or 45 $MNm^{-1.5}$. Thus based on the 'worst case scenario' fracture is expected, something readily concluded by inspection. However the probability of the toughness being less than the mean minus two (three) standard deviations value is 2.3% (0.135%) and likewise the probability of the applied stress intensity factor exceeding the mean plus two (three) standard deviations value is 2.3% (0.135%). The probability of the two events in combination is thus 5×10^{-4} (1.8×10^{-6}). It will be evident that if a large number of variables are involved, and a 'worst case' approach adopted, the level of conservatism becomes ludicrous.

D.3. Accurate Solution

For this simple case, an accurate solution can be obtained:-

Defining the safety margin (SM) as

$$SM = (S_m - L_m) / (\sigma_S^2 - \sigma_L^2)^{0.5}$$

where S_m = the mean strength
L_m = the mean load
σ_S = the standard deviation of strength
σ_L = the standard deviation of load

O'Connor (1991) showed that the reliability (proportion surviving) is given by
$\Phi(SM)$ where Φ is the cumulative distribution function for the normal distribution. In this case,

$$SM = (K_{1c,mean} - K_{app,mean}) / ((\text{s.d. for } K_{1c})^2 + (\text{s.d. for } K_{app})^2)^{0.5}$$

i.e.

$$(35 - 30) / (9 + 25)^{0.5} = 5 / 5.83 = 0.86$$

From standard statistical tables, $\Phi(0.86) = 0.8051$. Thus the reliability is 80% and we anticipate a 20% breakage rate.

D.4. The Monte Carlo Approach

This can be envisaged as a two stage approach. First a table is set up (Table2), with, for the purposes of this example, 20 rows, each corresponding to a range of standard deviation values above and below the mean which have an equal probability of occurrence (in this case 5%). The average standard deviation for the row is then calculated first in terms of standard deviations below and above the mean, then in terms of toughness and the applied s.i.f. The mean toughness and mean applied s.i.f. are then added to the appropriate standard deviation terms to give a table with 20 values of toughness (green), all equally likely to occur, and 20 values of the applied s.i.f. (red), also all equally likely.

In both cases the 20 values cover the full range expected with the proviso that the normal distribution is assumed to be truncated at ± 3 standard deviations.

It will be appreciated that whilst in this case, the toughness and applied s.i.f. have been assumed to be normally distributed, any distribution could be broken up in this way to produce a set of ranges each of which has an equal probability of occurrence.

In stage 2, a series of random numbers is generated, between 0 and 19 (as there are 20 rows in Table2). On the basis of one number a toughness selected, and the basis of the next, an applied stress intensity factor. One is thus directly simulating the real situation where these two will be brought together in a random combination. If the applied s.i.f. is greater or equal to the toughness, breakage is expected (Table3).

Three out of 40 load applications are predicted to result in breakage, i.e. 7.5%. This is not a particularly good estimate of the population breakage rate (20%). However as the number of iterations increases the breakage rate will converge on the true value. 500 or 1000 iterations may be a reasonable starting point.

Number	Lower end of range,s.d.'s	Upper end of range,s.d.'s	Average for range, s.d's	SD for K1c = 3	Add Mean = 35	SD for K_{appl} = 5	Add mean K = 30
0	-3	-1.65	-2.325	-6.975	28.025	-11.625	18.375
1	-1.65	-1.28	-1.465	-4.395	30.605	-7.325	22.675
2	-1.28	-1.04	-1.16	-3.48	31.52	-5.8	24.2
3	-1.04	-0.84	-0.94	-2.82	32.18	-4.7	25.3
4	-0.84	-0.67	-0.755	-2.265	32.735	-3.775	26.225
5	-0.67	-0.53	-0.6	-1.8	33.2	-3	27
6	-0.53	-0.39	-0.46	-1.38	33.62	-2.3	27.7
7	-0.39	-0.25	-0.32	-0.96	34.04	-1.6	28.4
8	-0.25	-0.13	-0.19	-0.57	34.43	-0.95	29.05
9	-0.13	0	-0.065	-0.195	34.805	-0.325	29.675
10	0	0.13	0.065	0.195	35.195	0.325	30.325
11	0.13	0.25	0.19	0.57	35.57	0.95	30.95
12	0.25	0.39	0.32	0.96	35.96	1.6	31.6
13	0.39	0.53	0.46	1.38	36.38	2.3	32.3
14	0.53	0.67	0.6	1.8	36.8	3	33
15	0.67	0.84	0.755	2.265	37.265	3.775	33.775
16	0.84	1.04	0.94	2.82	37.82	4.7	34.7
17	1.04	1.28	1.16	3.48	38.48	5.8	35.8
18	1.28	1.65	1.465	4.395	39.395	7.325	37.325
19	1.65	3	2.325	6.975	41.975	11.625	41.625

Table 2

Random No. 1	K_{1c}	Random No.2	K_{app}	
15	37.265	7	28.4	
19	41.975	12	31.6	
14	36.8	11	30.95	
12	35.96	6	27.7	
5	33.2	18	37.325	Break
6	33.62	3	25.3	
7	34.04	8	29.05	
4	32.735	11	30.95	
3	32.18	0	18.375	
1	30.605	1	22.675	
12	37.265	15	33.775	
0	28.025	18	37.325	Break
1	30.605	0	18.375	
6	33.62	19	41.625	Break
11	35.57	8	29.05	
5	33.2	11	30.95	
18	39.395	8	29.05	
10	35.195	16	34.7	
6	33.62	13	32.3	
8	34.43	15	33.775	
11	35.57	3	25.3	
13	36.38	16	34.7	
16	37.82	7	28.4	
15	37.265	12	31.6	
13	36.38	15	33.775	
3	32.18	6	28.4	
12	35.96	16	34.7	
9	34.805	8	29.05	
19	41.975	10	30.325	
10	35.195	8	29.05	
2	31.52	6	27.7	
6	33.62	1	22.675	
4	32.735	6	27.7	
6	33.62	11	30.95	
10	35.195	13	32.3	
10	35.195	9	29.675	
10	35.195	15	33.775	
15	37.265	2	24.2	
14	36.8	14	33	
7	34.04	2	24.2	

Table 3 Randomly compared toughness and applied s.i.f. values